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STUDENT SPACE PROGRAM

A Miniaturized Satellite Attitude Determination and Control System with Autonomous Calibration Capabilities

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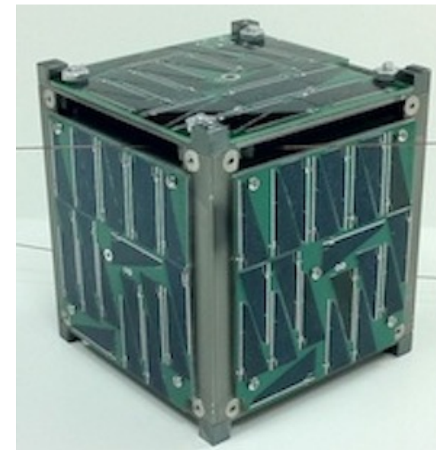
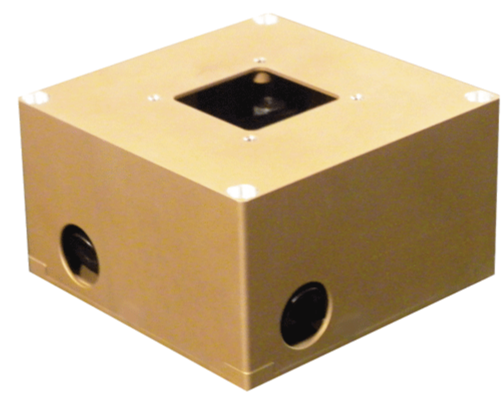
Introduction

- ADACS designed for CubeSats
- CubeSats generally range in size from 1U to 3U (10x10x(10-30) cm)
  - Relatively cheap and easy to build
  - Highly capable due to miniaturized technology
- Require attitude control systems to properly orient science instruments and antennas





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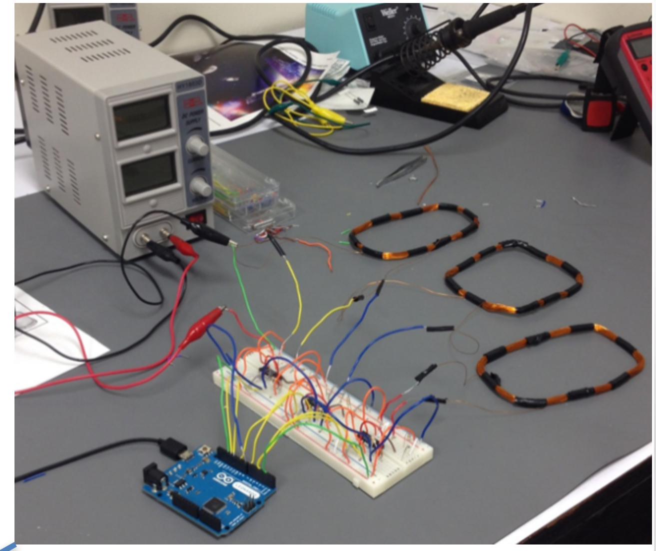
Sensors and Actuators Trade Study

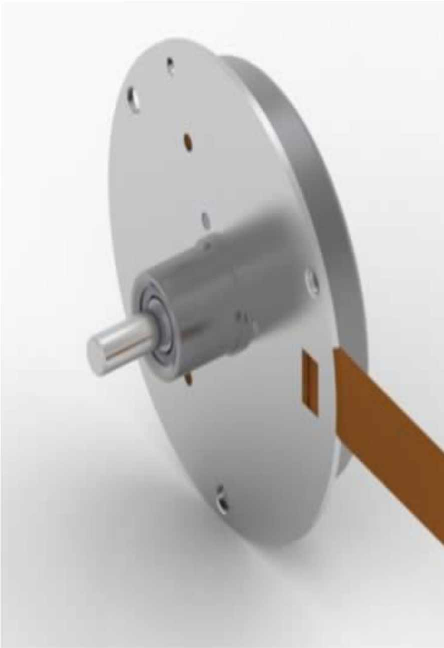
10 is most desirable, 1 is least desirable

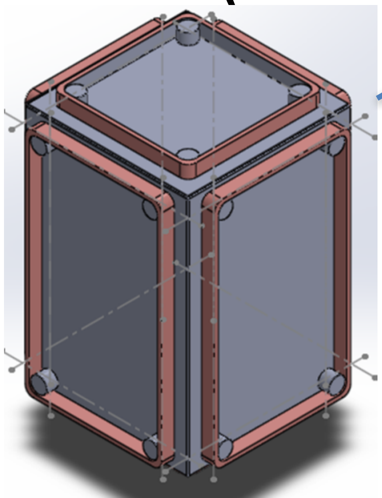
Name	Cost	Complexity	Legacy	Accuracy	Totals
Magnetometer	8	8	10	6	32
Horizon Sensor	6	7	8	8	29
Sun Sensor	10	9	10	2	31
Star Tracker	1	3	5	10	19
Rate Gyros	10	8	8	5	31
Magnetorquers	10	6	10	5	31
3-axis Reaction Wheels	7	6	8	9	30
Gravity Boom	10	10	1	2	23
Aerodynamic Stabilization	10	10	2	5	27

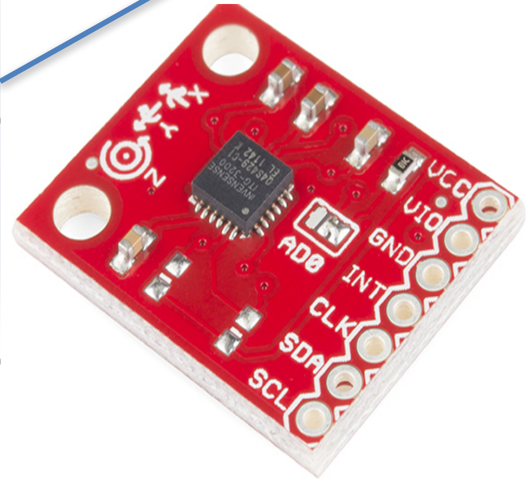
Sensors and Actuators

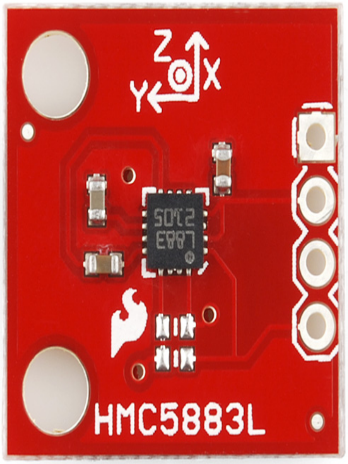
- Reaction Wheels (3)
- Magnetorquers (3)
- Magnetometer (3-axis)
- Rate gyros (3-axis)
- Position Sensitive Detectors (PSDs)











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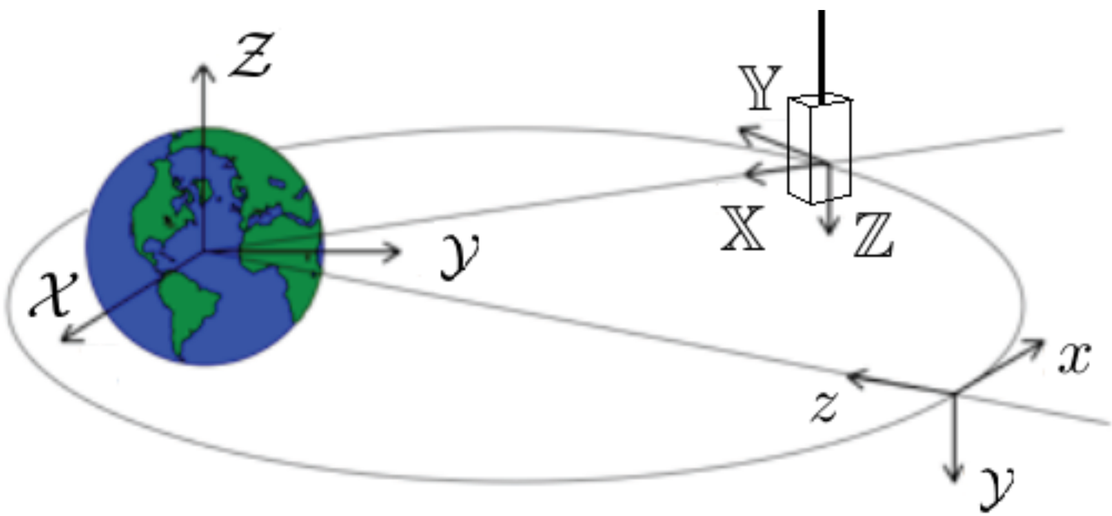
Coordinate Frames

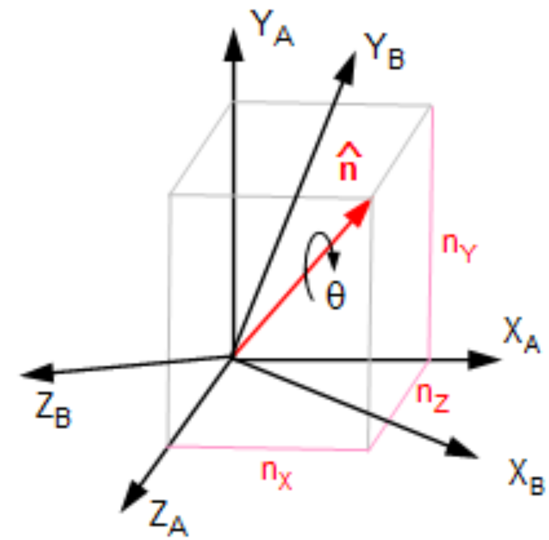
- Earth Fixed Inertial Frame – Origin at Earth center, x-axis toward vernal equinox, z-axis through North Pole
- Orbital Frame – Origin at satellite cm, x-axis aligned with velocity, z-axis toward Earth
- Satellite Body Frame – Origin at satellite cm and moves with the satellite



Coordinate Frame Transformation

- Direction Cosine Matrix

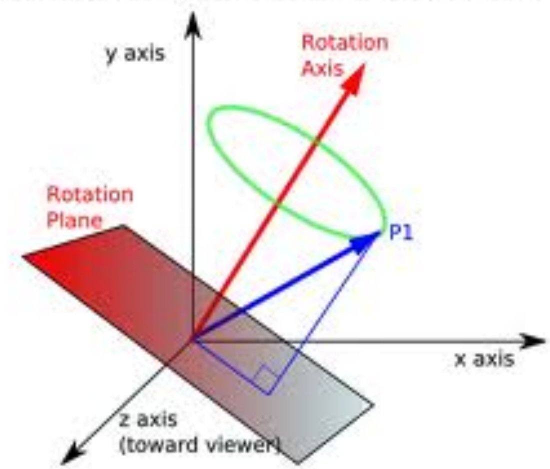
$$\begin{bmatrix} r_{x2} \\ r_{y2} \\ r_{z2} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} r_{x1} \\ r_{y1} \\ r_{z1} \end{bmatrix}$$



- Quaternion
  - Rotation vector ($\vec{n}$) plus rotation about vector (θ)

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \cos(\theta/2) \\ n_x \sin(\theta/2) \\ n_y \sin(\theta/2) \\ n_z \sin(\theta/2) \end{bmatrix}$$

A quaternion, as devised by William Hamilton, can be used to transform one 3-D vector into another



Attitude Vectors

- Sun vector in body frame from PSDs
- Magnetic field vector in body frame from magnetometer
- Magnetic field vector in orbital frame from IGRF model
- Sun vector in orbital frame from heliocentric orbit propagator
- Angular velocity vector (in body frame) from rate gyros

Attitude Determination

- Defined as direction cosine matrix between body frame and orbital frame
- Triad algorithm gives DCM based on two vectors known in both frames

$$\begin{bmatrix} \vec{R}_1 & \vec{R}_2 & (\vec{R}_1 \times \vec{R}_2) \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} \vec{r}_1 & \vec{r}_2 & (\vec{r}_1 \times \vec{r}_2) \end{bmatrix}$$

- DCM to quaternion conversion

$$q_0^{\ 2} = \frac{tr(A)+1}{4} = \frac{a_{11}+a_{22}+a_{33}+1}{4}$$

$$q_1 = \frac{a_{32}-a_{23}}{4q_0}$$

$$q_2 = \frac{a_{13}-a_{31}}{4q_0}$$

$$q_3 = \frac{a_{21}-a_{12}}{4q_0}$$

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Error Quaternion

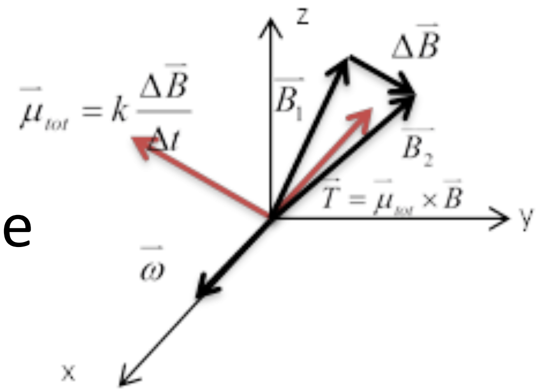
- Relates current orientation to desired orientation
  - Used in control algorithm
- Calculated based on measured and desired quaternions (q_m and q_d)

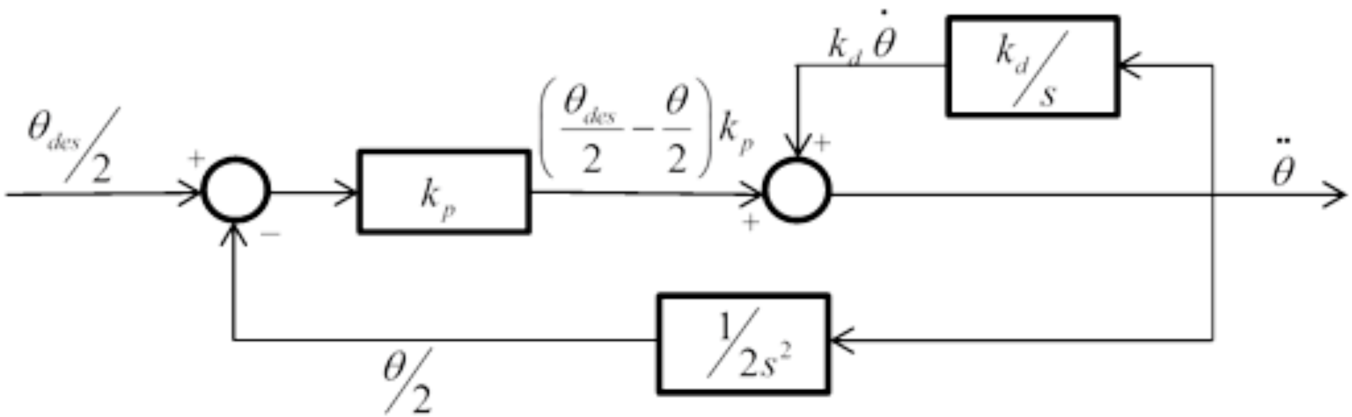
$$L_d = \begin{bmatrix} q_{0_d} & q_{1_d} & q_{2_d} & q_{3_d} \\ -q_{1_d} & q_{0_d} & q_{3_d} & -q_{2_d} \\ -q_{2_d} & -q_{3_d} & q_{0_d} & q_{1_d} \\ -q_{3_d} & q_{2_d} & -q_{1_d} & q_{0_d} \end{bmatrix}$$

$$q_e = L_d q_m$$

Control Algorithm

- B-Dot de-tumble algorithm
  - Minimize angular velocity by torqueing opposite to direction of angular velocity vector
  - $\vec{\mu} = k\dot{\vec{B}}$
  - $\vec{T} = \vec{\mu}\times\vec{B}$
- Inverse Dynamics PD Controller for steady state pointing
  - Proportional (error) and derivative terms
  - Analytical gain estimation from block diagram





$$T(s) = \frac{k_p}{s^2 - sk_d + \frac{k_p}{2}}$$

Inverse Dynamics PD Controller

- Takes satellite dynamics into account
  - Ensures stability in all conditions
- PD law determines desired angular accelerations
- Equations of motion used to calculate required torques
- Gains independent of inertia properties

$$\begin{bmatrix}\alpha_x\\ \alpha_y\\ \alpha_z\end{bmatrix} = k_p\begin{bmatrix}q_{e1}\\ q_{e2}\\ q_{e3}\end{bmatrix} + k_d\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}$$

$$\sum M = \frac{dH}{dt} = 0$$

$$\begin{bmatrix}T_x\\ T_y\\ T_z\end{bmatrix}_m = I_w\begin{bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{bmatrix}_w = \left(\begin{bmatrix}I_{11} & 0 & 0\\ 0 & I_{22} & 0\\ 0 & 0 & I_{33}\end{bmatrix}_{sc}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}_{sc} + I_w\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix}_w\right)\times\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix}_{sc} - \begin{bmatrix}I_{xx} & 0 & 0\\ 0 & I_{yy} & 0\\ 0 & 0 & I_{zz}\end{bmatrix}_{sc}\begin{bmatrix}\alpha_x\\ \alpha_y\\ \alpha_z\end{bmatrix}_{sc}$$

Drawbacks of Direct Linear PD Controller

- Torques calculated directly from controller

$$\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}_m = \begin{bmatrix} k_{px} q_{e1} \\ k_{py} q_{e2} \\ k_{pz} q_{e3} \end{bmatrix} + \begin{bmatrix} k_{dx} \omega_x \\ k_{dy} \omega_y \\ k_{dz} \omega_z \end{bmatrix}$$

  – Controller does not take wheel angular momentum into account
    - Can result in instability

  – Controller gains dependent on satellite inertia properties

  – Separate gain value needed for each parameter

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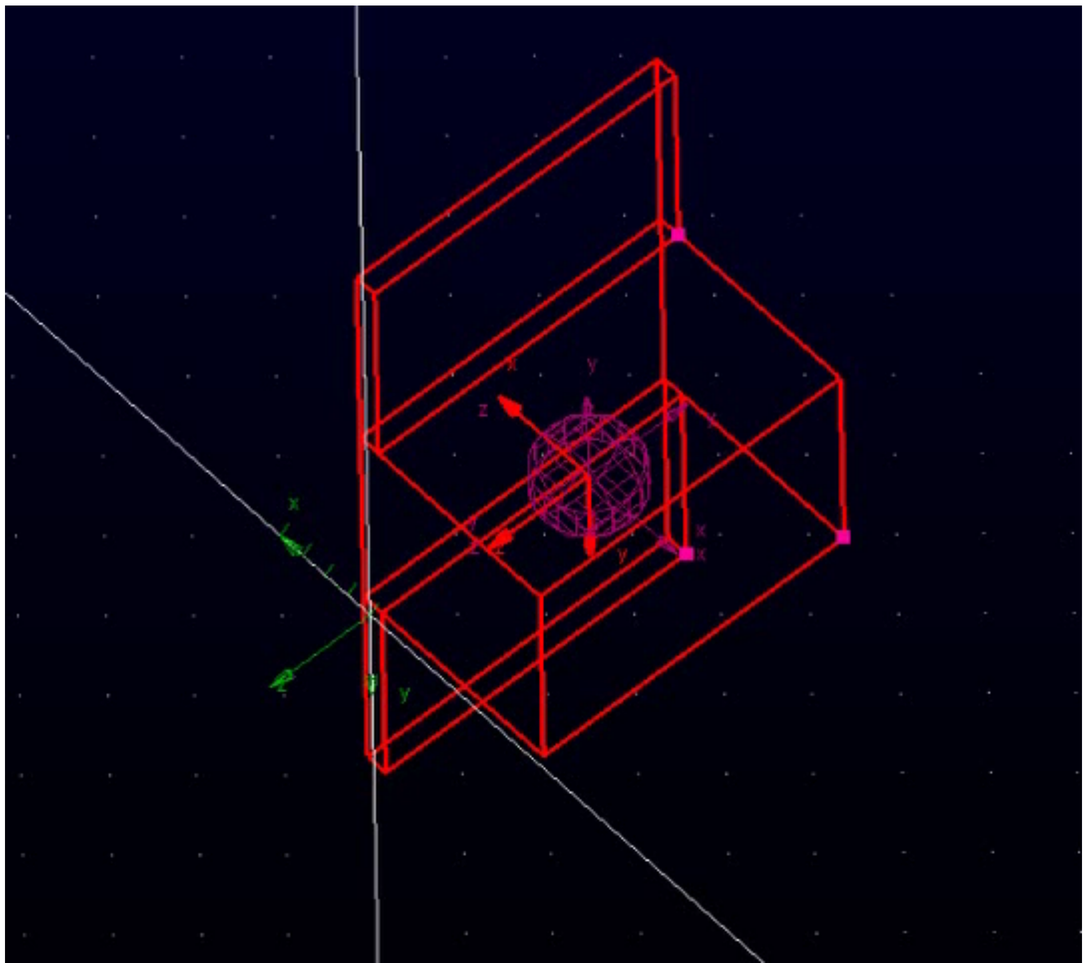
Instability of Direct PD Control when Angular Momentum is Large



Inverse Dynamics PD Controller Performance

- Desired angular accelerations achieved perfectly
- Performance independent of initial conditions and system inertia properties
- System exhibits a stable oscillatory equilibrium
  - Frequency and amplitude of oscillations decreases as system becomes more overdamped
  - Adding integral term does not mitigate oscillations
  - More work needed to determine exact cause for oscillations

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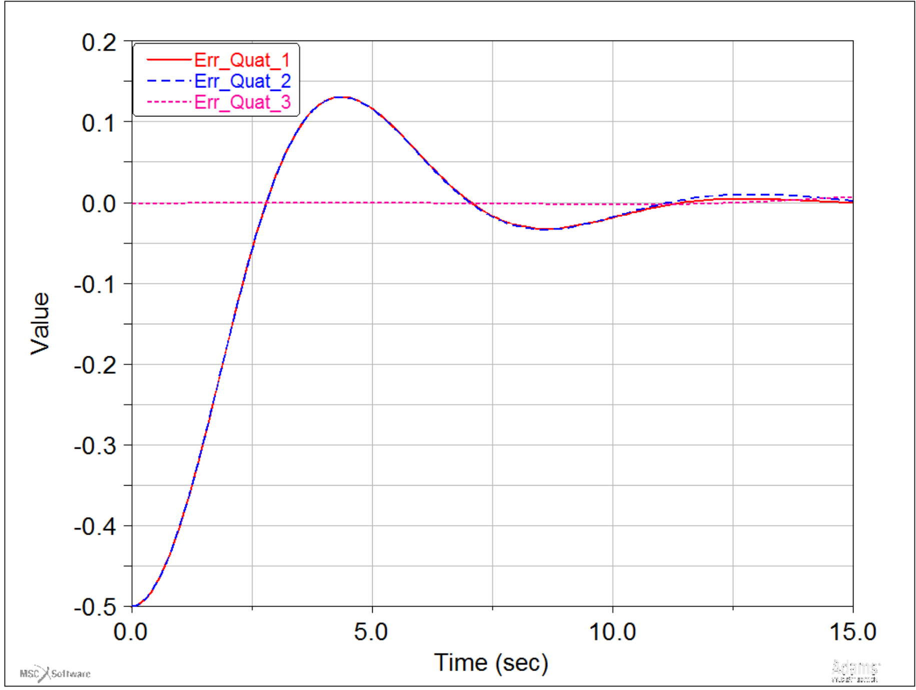
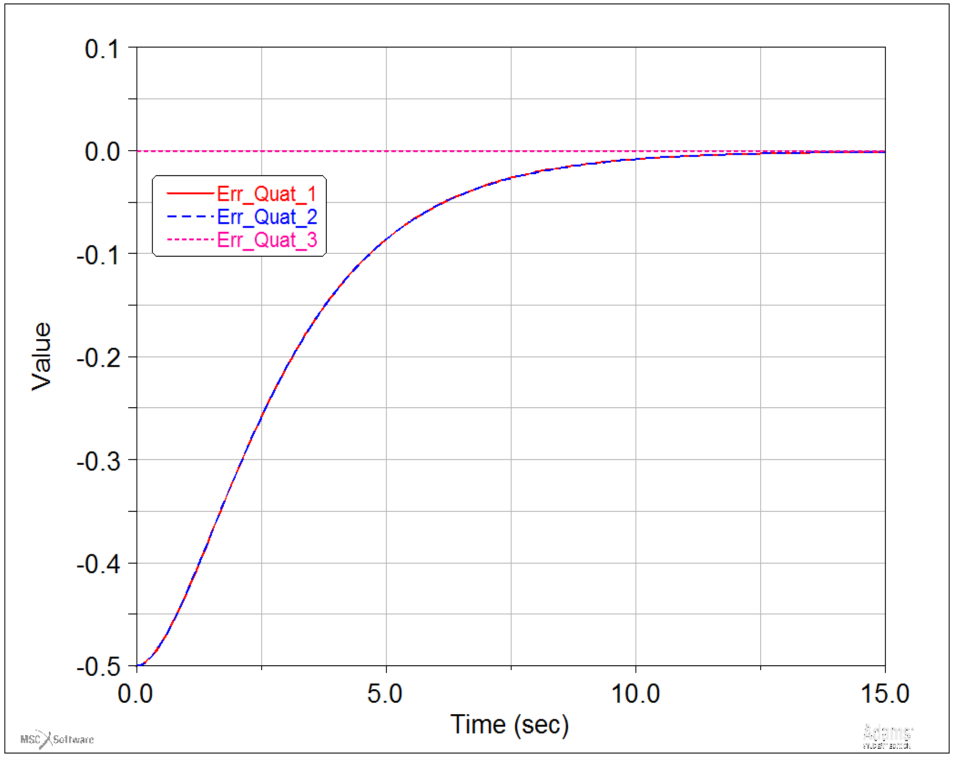
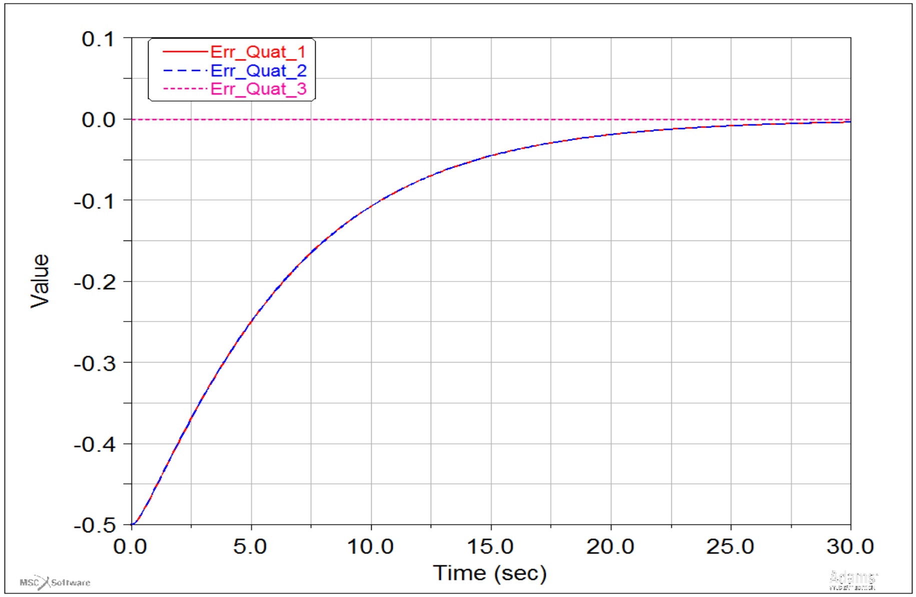
Results $\overrightarrow{\omega_0} = (0, 0, 0)$



Underdamped Case (K_p=1, k_d=-.5)



Critically Damped Case (K_p=1, k_d=-1.4)



Overdamped Case (K_p=1, k_d=-4)

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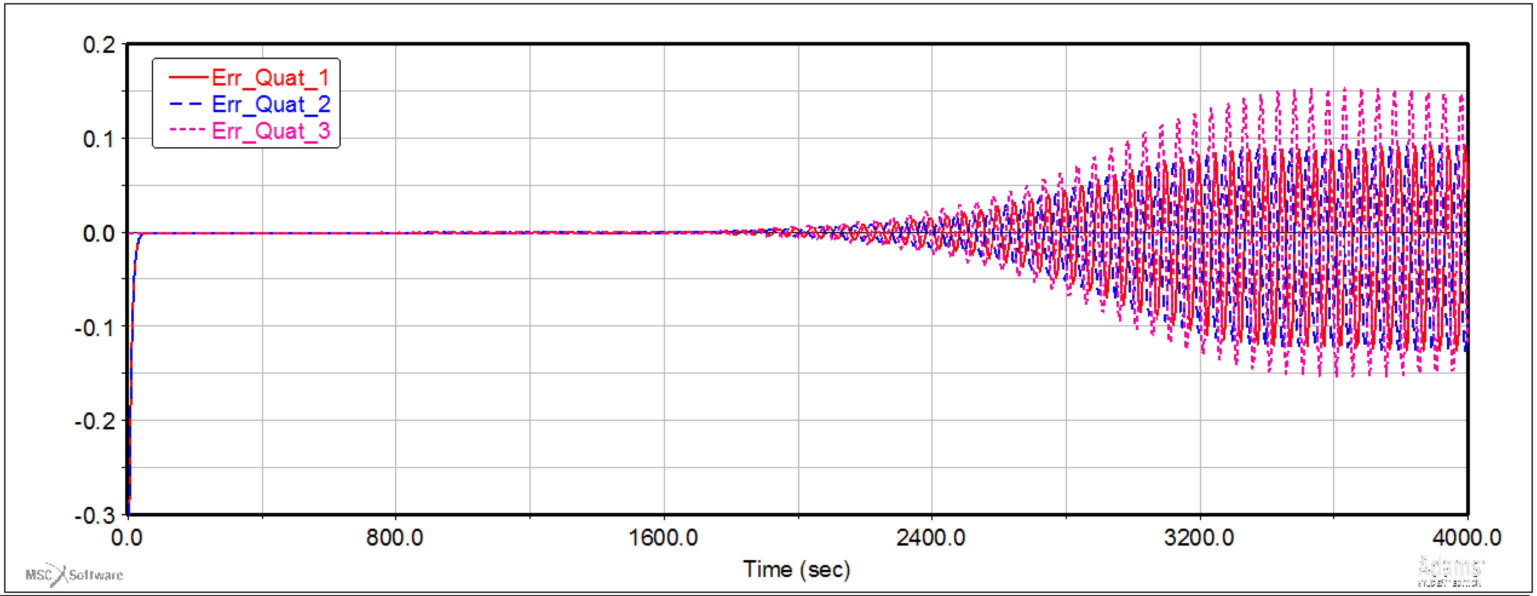
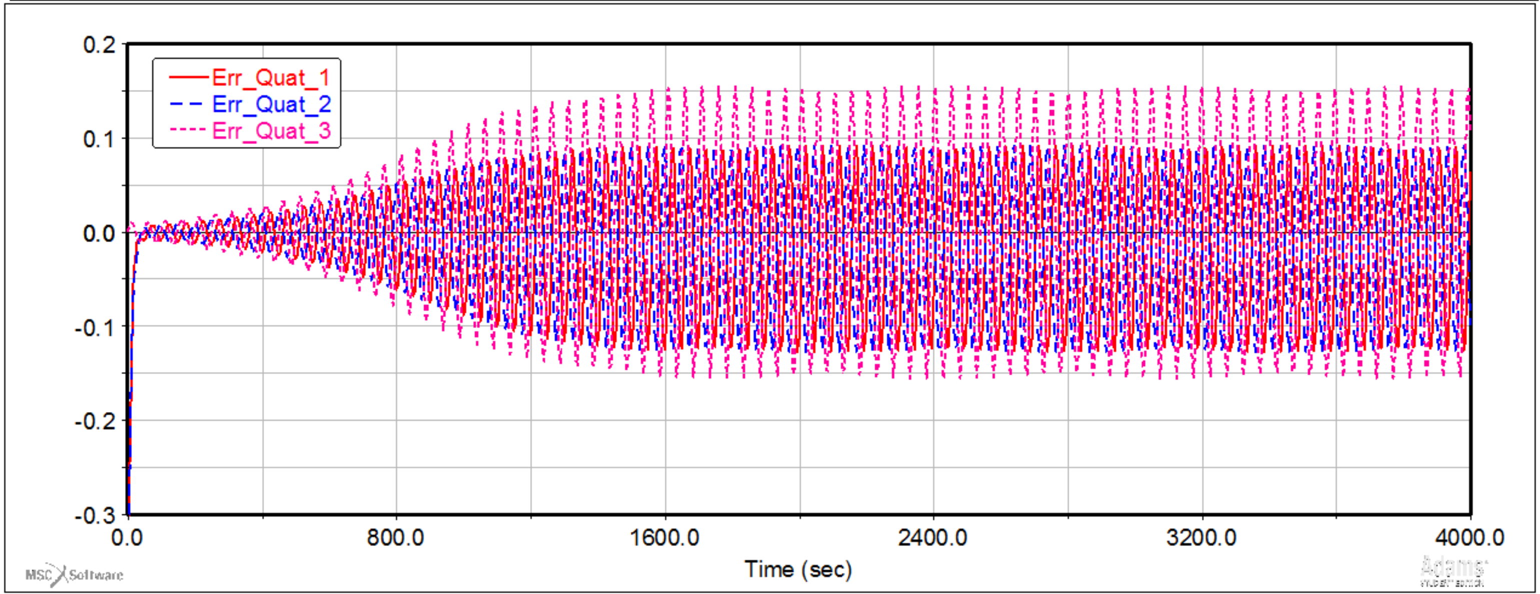
Simulations with Kp = 1 and Kd = -4

$\overrightarrow{\omega_0} = (0,0,0)$



$\overrightarrow{\omega_0} = (.5,.1,.7)$



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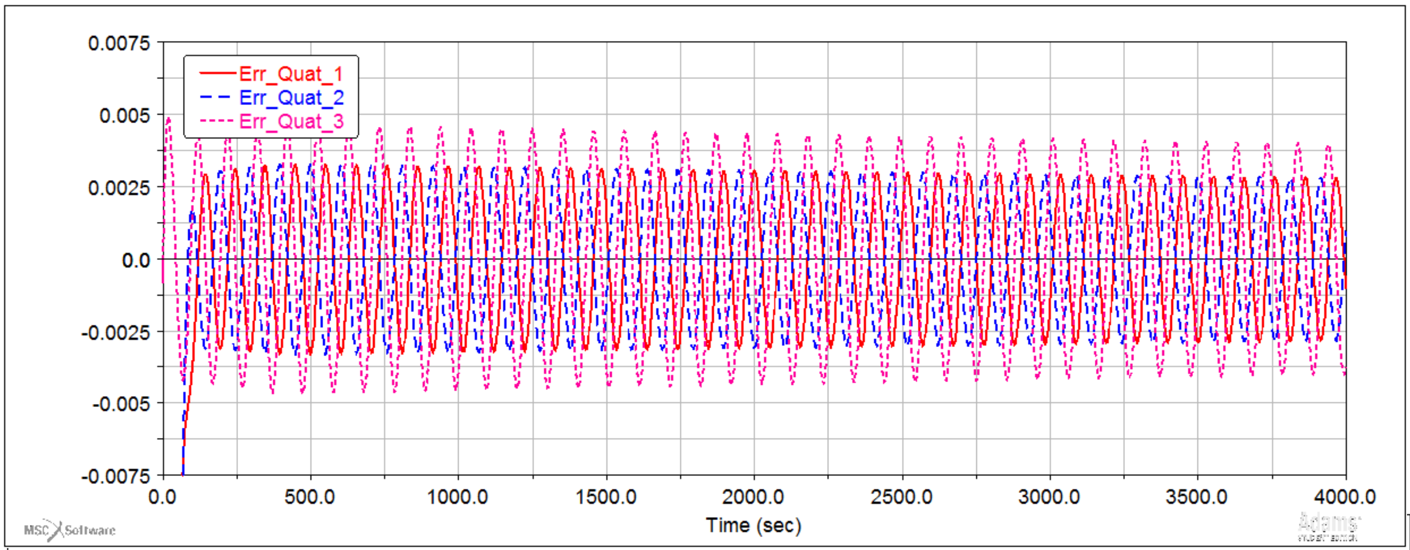
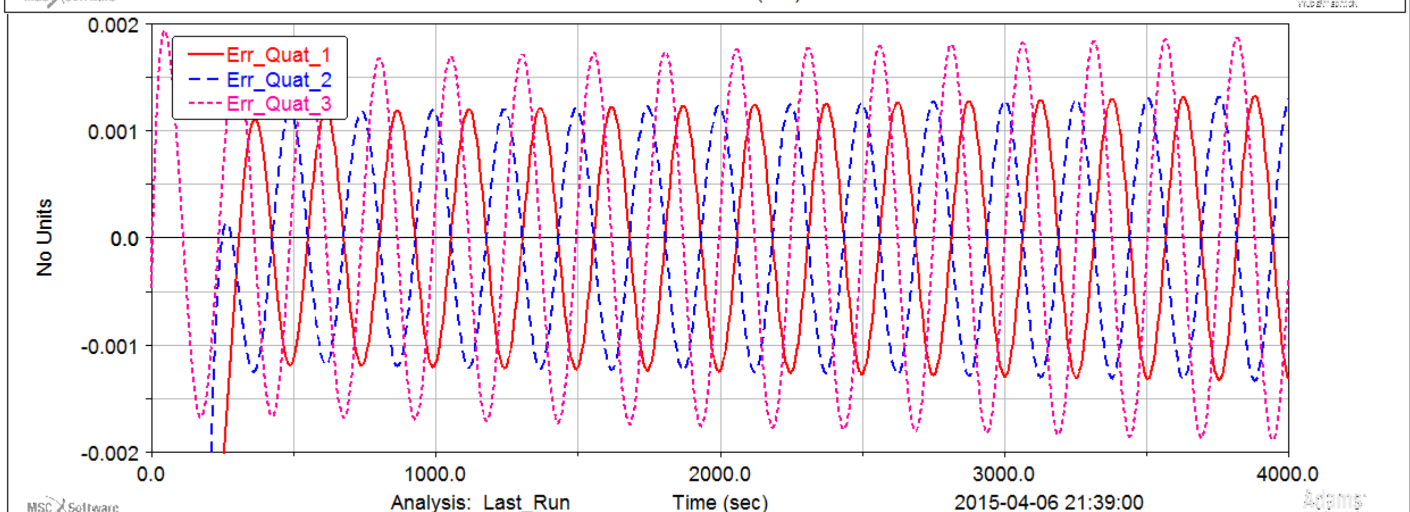
Simulations with $\overrightarrow{\omega_0} = (.5, .1, .7)$

$K_p = 1$
$k_d = -8$



$K_p = 1$
$k_d = -20$



Minimizing Oscillations

- Damping ratio $\zeta = \left|\frac{k_d}{2\sqrt{k_p}}\right|$
- Increase both k_d and k_p such that system is heavily overdamped
  - High damping minimizes oscillations
  - High k_p means system will get to desired orientation rapidly

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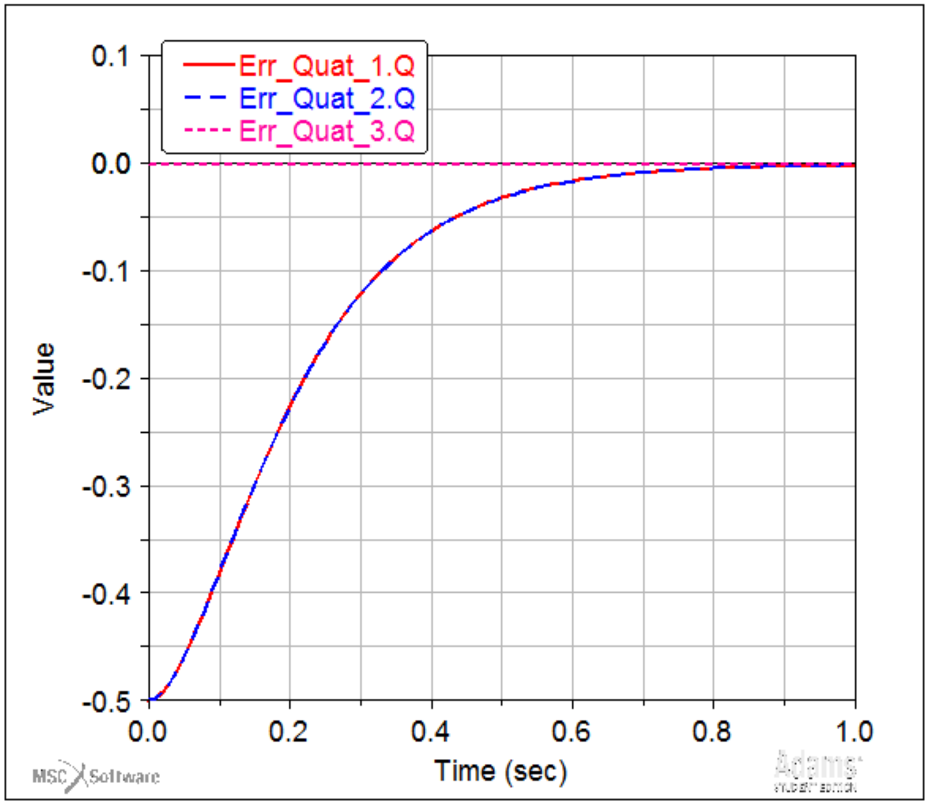
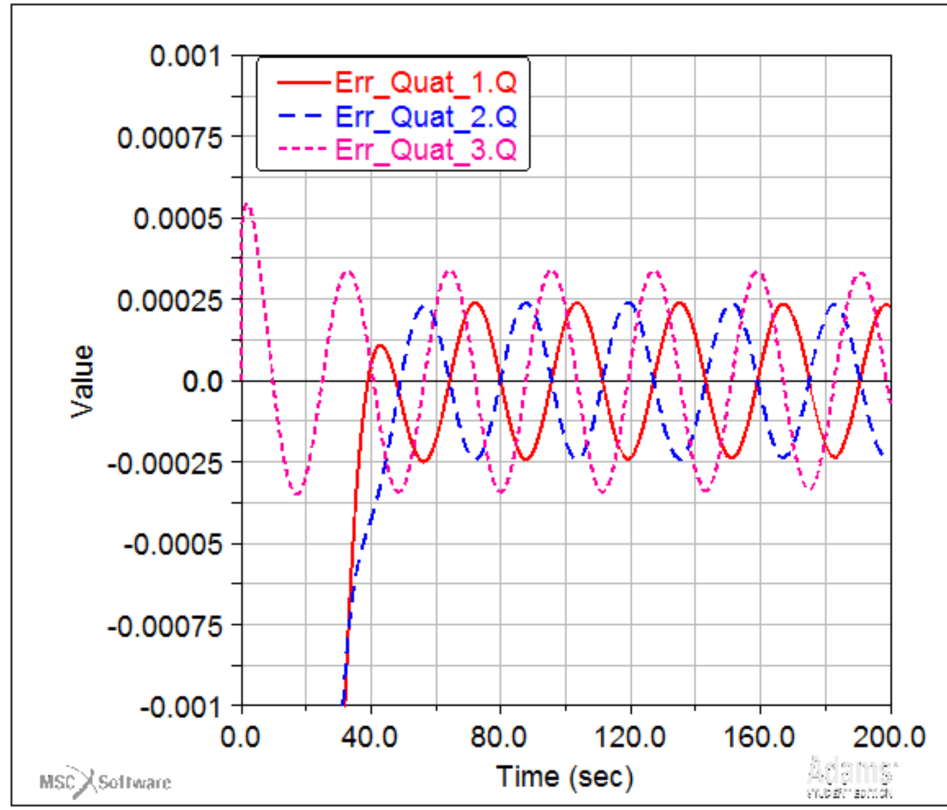
Simulations with K_p = 200





K_d = -20 and $\overrightarrow{\boldsymbol{\omega_0}} = (\mathbf{0}, \mathbf{0}, \mathbf{0})$

K_d = -500 and $\overrightarrow{\boldsymbol{\omega_0}} = (.\mathbf{5}, .\mathbf{1}, .\mathbf{7})$

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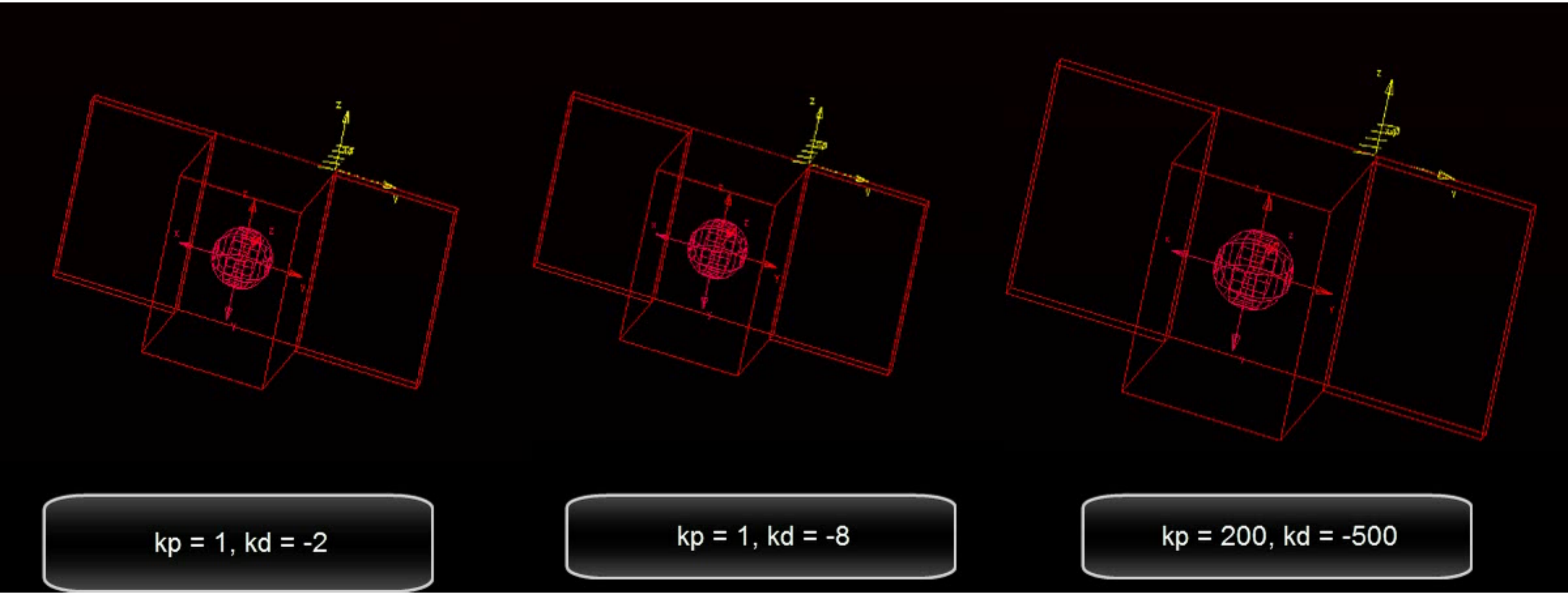
Simulations with Different Gains and $\overrightarrow{\omega_0} = (.5, .1, .7)$



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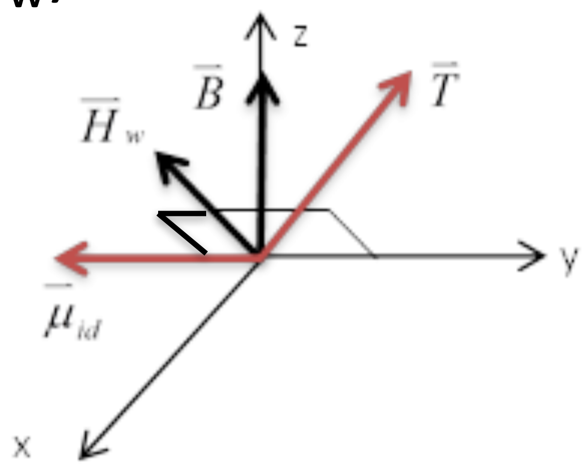
Active Momentum Dumping

- Magnetorquers constantly work to reduce wheel angular momentum (H_w)

$$\vec{H}_w = I_w \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

$$\vec{\mu}_{id} = k * \left(\vec{H}_w \times \vec{B} \right)$$

$$\vec{T} = \vec{\mu}_{id} \times \vec{B} = k * \left(\vec{H}_w \times \vec{B} \right) \times \vec{B}$$



Autonomous In-Flight Calibration

- Estimate moments of inertia and angular velocity based on kinematic response

$$\sum M = \frac{d\vec{H}}{dt} + \vec{\omega}_{sc} \times \vec{H}$$

  – Calibrate to correct bias error in rate gyros.

$$I_s \alpha_s = -I_w \alpha_w$$

$$\omega_x = \frac{I_w \omega_y \omega_1 - I_w \dot{\omega}_3 - I_{zz} \dot{\omega}_z}{\left(I_{yy} - I_{xx}\right)\omega_y + I_w \omega_3}$$

$$\omega_z = \frac{I_w \omega_x \omega_3 + I_w \dot{\omega}_2 - I_{yy} \dot{\omega}_y}{\left(I_{xx} - I_{zz}\right)\omega_x + I_w \omega_1}$$

$$\omega_y = \frac{I_w \omega_z \omega_2 - I_w \dot{\omega}_1 - I_{xx} \dot{\omega}_x}{\left(I_{zz} - I_{yy}\right)\omega_z + I_w \omega_3}$$

Estimated Cost

- Four Positions Sensitive Detectors - $1000 total
- Three Magnetorquers - $100 total
- Three Reaction Wheels - $300 total
- Electronic Speed Controller - $50
- Magnetometer - $100
- Rate gyros - $100
- Microcontroller and PCB - $200
- Total - $1850

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Questions?

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