Texas Spacecraft Lab

A Software Tool for CubeSat Mission Risk Estimating Relationships

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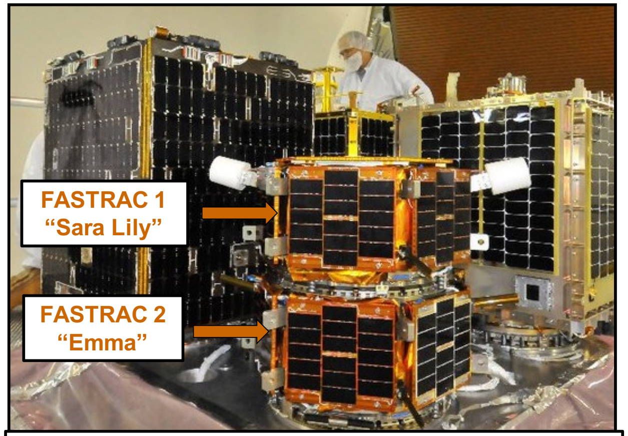
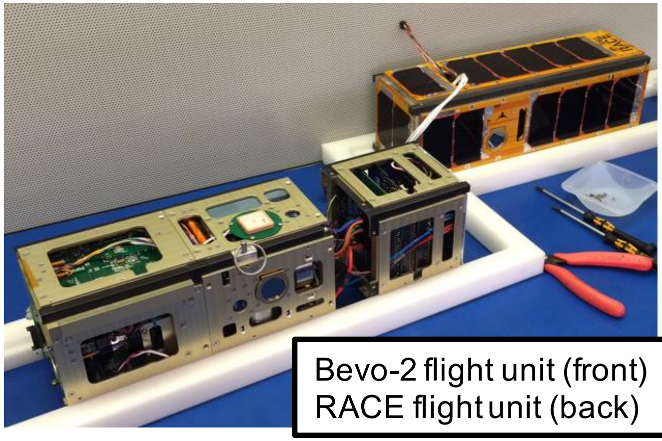
Texas Spacecraft Laboratory

Texas Spacecraft Lab

- Labs located on campus in WRW building, 4th floor
- Entirely student-led with a faculty PI (Dr. Glenn Lightsey)
- Current flight experience:
  - FASTRAC nanosatellite (25 kg each), still operational, launched in Nov. 2010
  - Bevo-1/Paradigm (1U) launched in July 2009
- On the horizon:
  - RACE (3U) w/ JPL
    - Delivered spacecraft bus for radiometer mission, Mar 2014
    - To be flown via NanoRacks, October 2014
  - Bevo-2 (3U) w/ NASA-JSC & Texas A&M
    - Delivery to NASA in June 2014
    - To be flown via NanoRacks
  - ARMADILLO (3U) w/ Baylor University
    - University Nanosatellite Program winner, Jan. 2013
    - Selected for ELaNa in Spring 2012 (to be manifested)
  - INSPIRE (3U) w/ JPL
    - Providing thruster; collaboration with other organizations
    - To be flown on interplanetary trajectory



FASTRAC Satellites Mated on STP-S26.
Credit: U.S. Air Force photo by Lou Hernandez



Bevo-2 flight unit (front)
RACE flight unit (back)

Talk Overview

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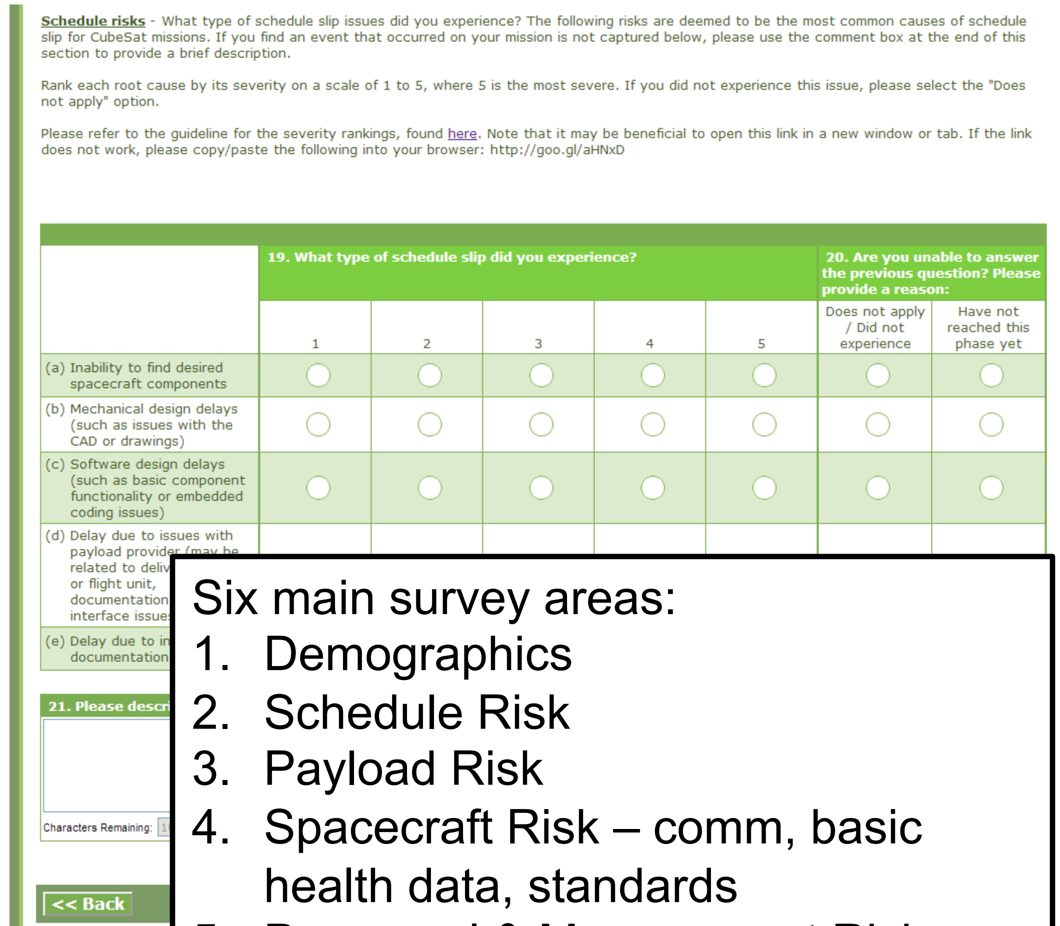
- CubeSat Mission Risk Survey – Results!
- Risk Analysis Tool
  - Mathematical overview
  - Tool overview
  - How to obtain a copy (Feedback wanted!)
- Decision Advisor


Schedule risks - What type of schedule slip issues did you experience? The following risks are deemed to be the most common causes of schedule slip for CubeSat missions. If you find an event that occurred on your mission is not captured below, please use the comment box at the end of this section to provide a brief description.

Rank each root cause by its severity on a scale of 1 to 5, where 5 is the most severe. If you did not experience this issue, please select the "Does not apply" option.

Please refer to the guideline for the severity rankings, found here. Note that it may be beneficial to open this link in a new window or tab. If the link does not work, please copy/paste the following into your browser: http://goo.gl/aHNxD

	19. What type of schedule slip did you experience?					20. Are you unable to answer the previous question? Please provide a reason:	
	1	2	3	4	5	Does not apply / Did not experience	Have not reached this phase yet
(a) Inability to find desired spacecraft components	○	○	○	○	○	○	○
(b) Mechanical design delays (such as issues with the CAD or drawings)	○	○	○	○	○	○	○
(c) Software design delays (such as basic component functionality or embedded coding issues)	○	○	○	○	○	○	○
(d) Delay due to issues with payload provider (may be related to deliv… or flight unit, documentation… interface issue…							
(e) Delay due to in… documentation…							

21. Please descr…

Characters Remaining:

<< Back

Six main survey areas:

1. Demographics
2. Schedule Risk
3. Payload Risk
4. Spacecraft Risk – comm, basic health data, standards
5. Personnel & Management Risk
6. Cost Risk

Survey Data Results

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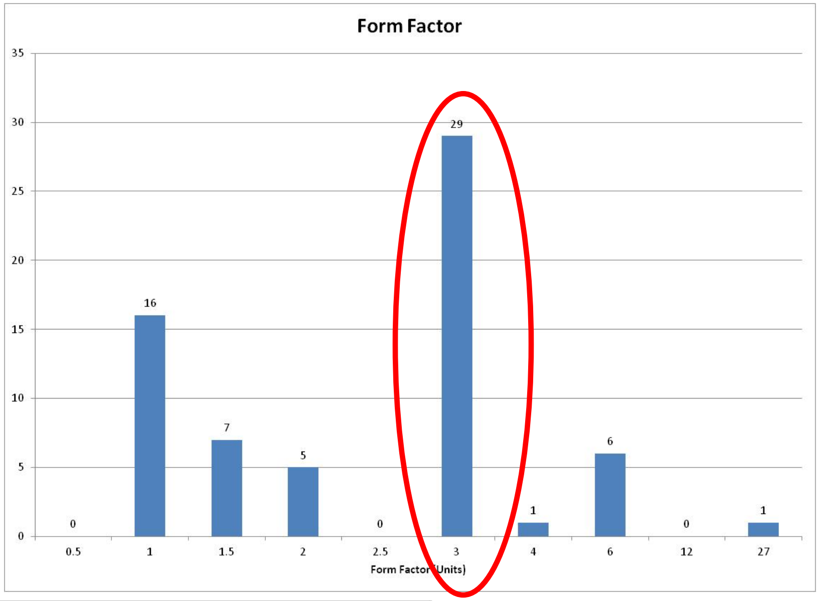
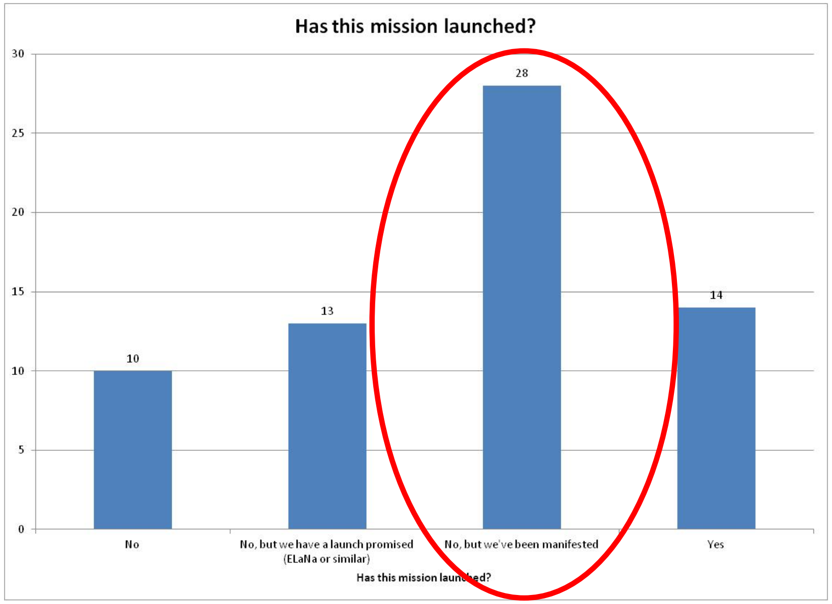
- THANK YOU!
- Collected responses April – November 2013
- 65 CubeSat responses
- 52 unique and valid missions used for high-level analysis
- 3 outliers were removed for regression analysis





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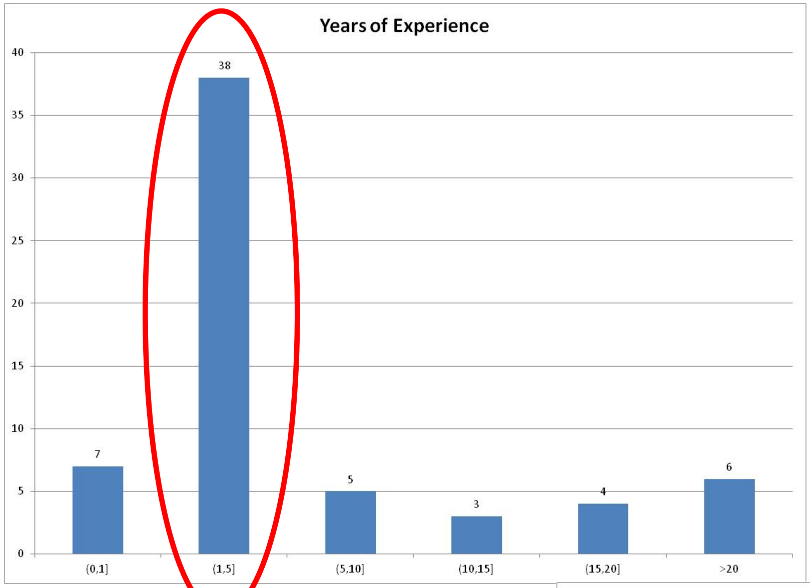
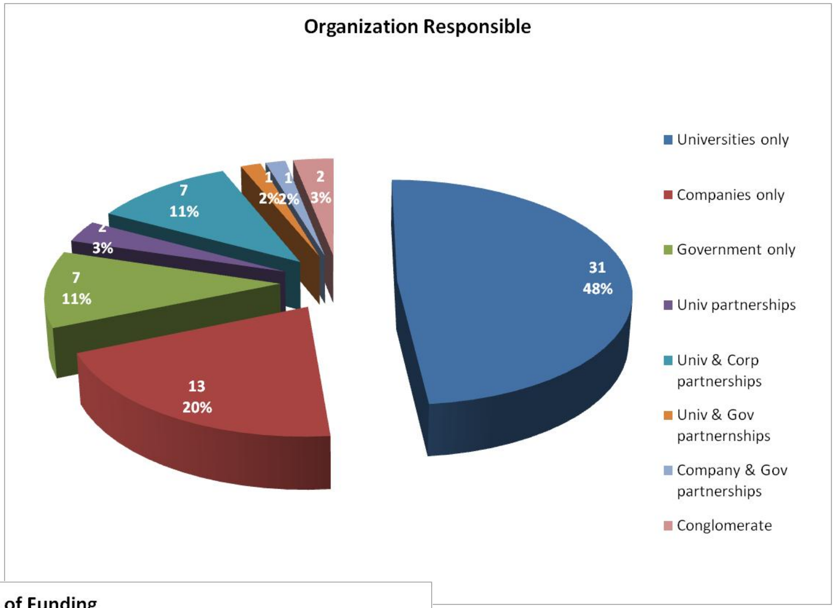
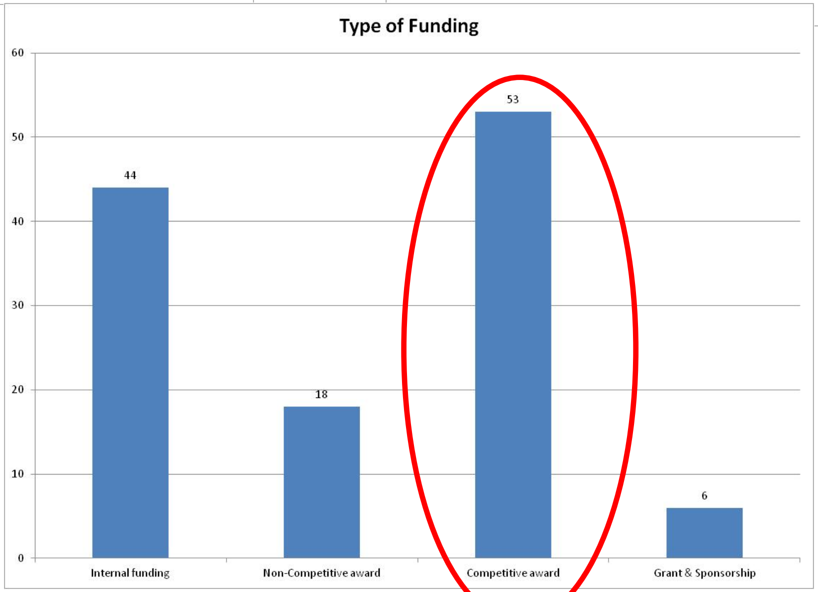
Aerospace Engineering and Engineering Mechanics

Survey Data Results







Survey Risk Results

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Top Ten Risk Events as identified by survey responses:

1. Software design delay
2. Attrition or turnover of team members
3. Mechanical design delay
4. Incomplete understanding of the projected total mission cost
5. Inability to find desired spacecraft components
6. Sudden loss of crucial team members
7. Delay due to inadequate documentation
8. Loss of information
9. Lack of sufficient training for team members completing flight qualification necessary tasks
10. Delay due to issues with payload provider

All events in the top ten have to do with personnel, schedule, or cost

Risk Tool – Mathematical Overview

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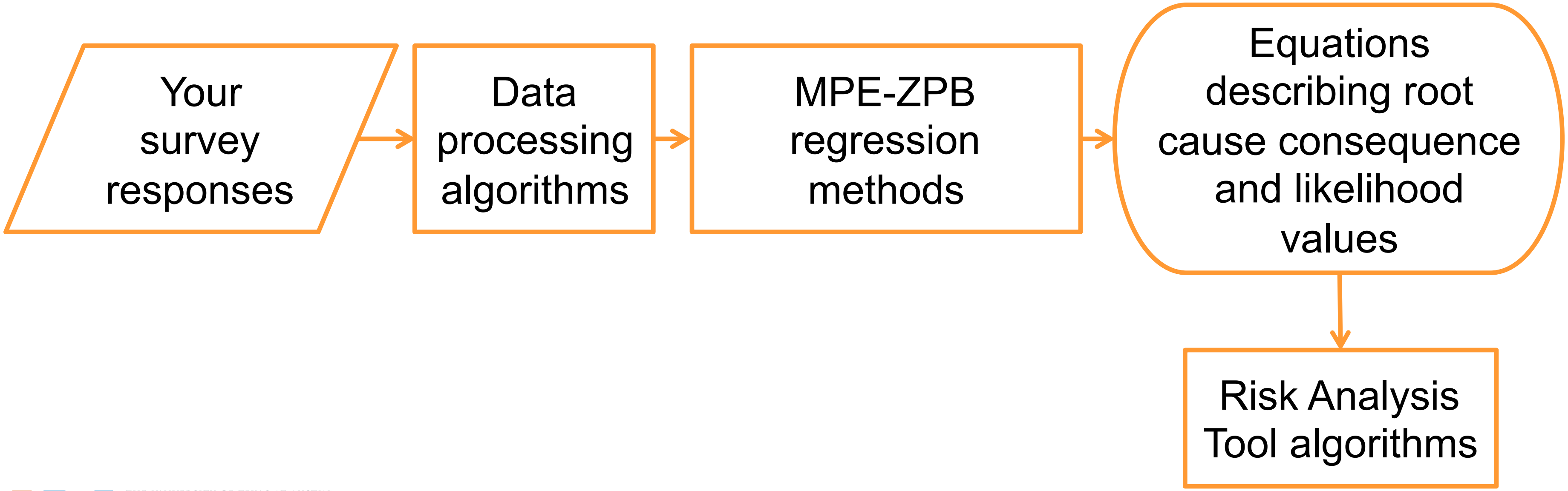
- Used General Error Regression, Minimum Percentage Error – Zero Percentage Bias (MPE-ZPB)
  - Similar regression method used by USCM and SSCM cost models
  - Comparable to Least Squares, but multiplicative error
- 12 Function Forms tested
- Used Excel VBA and Solver



Risk Tool V1.1 – Inputs Page

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Factors of interest in regression analysis

Parameter	Input	Actual or Predicted?	Description
Form factor	3		*Enter a numeric value corresponding to the number of U's your spacecraft design uses (e.g. 3U would be entered as "3")*
Mass	4		*Enter a numeric value of the mass limit (in kg)*
Launched?	No, but we have a launch promised (ELaNa or similar)		*Select an answer using the drop-down menu: Yes, the s/c has launched; No, but we've been manifested; No, but we have a launch promised (ELaNa or similar); No, we have not been manifested or given a promise of a launch*
Launch Date	2014		*Give the date of the launch; If the s/c has yet to be launched, give the projected date. (Can be in MM/DD/YYYY or MM/YYYY or YYYY format)*
Months in Development	7	Actual	*Enter a numeric value corresponding to the number of months in s/c design and development, including everything up until flight integration; Indicate whether this value is actual or predicted*
Months in Integration	4	Actual	*Enter a numeric value corresponding to the number of months taken for s/c integration; Indicate whether this value is actual or predicted*
Months in S/C Functional Testing	7	Predicted	*Enter a numeric value corresponding to the number of months spent on integrated s/c testing at the organization level, including functional testing; Indicate whether this value is actual or predicted*
Months in S/C Environmental Testing	5	Predicted	*Enter a numeric value corresponding to the number of months spent on necessary testing to satisfy launc thermal vac, vib tables, and mas value is actual or predicted*
Months S/C is awaiting launch	3	Predicted	*Enter a numeric value correspon spacecraft was "on the shelf" wa completed; Indicate whether this value is actual or predicted*
Months S/C is in operations	6	Predicted	*Enter a numeric value corresponding to the number of months the spacecraft was operational in orbit; Indicate whether this value is actual or predicted*
Milestone	LVINT		*Enter the name of the milestone for which these numbers reflect the status*

Options:

- Calculate L-C values for Milestone 1
- Calculate L-C values for Milestone 2
- Calculate L-C values for Milestone 3
- Clear Error Messages and Warnings
- Clear Milestone 1 Values
- Clear Milestone 2 Values
- Clear Milestone 3 Values

Macro buttons will calculate the L-C values for multiple milestones

Indicate whether values are actual or predicted

Risk Tool V1.1 – Outputs Page

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		Milestone 1		Milestone 2		Milestone 3	
Mission Risk	**Root Cause**	**Consequence value**	**Likelihood value**	**Consequence value**	**Likelihood value**	**Consequence value**	**Likelihood value**
Schedule		3.31804782	4.487230123	.290284849	4.471377851	3.17733186	4.535742296
	1. Inability to find desired spacecraft components	2.212231943	4.352609117	2.212231943	4.392936827	2.212231943	4.557244851
	2. Mechanical design delays (such as issues with the CAD or drawings)	2					
	3. Software design delays (such as basic component functionality or embedded coding issues)	3.941404598	4.774450701	3.869117987	4.699644896	3.431920442	4.693574931
	4. Delay due to issue with payload provider (may be related to delivery of		242310987	3.564926097	4.293131741	3.564926097	4.405439749
			960906613	2.375918631	4.055460527	3.130304667	4.242332588
P			478869868	3.3175176	4.538279758	3.435401292	4.709801103
	payload and spacecraft bus	3.319286913	4.716094717	3.021747664	4.713521567	2.931091293	4.853501468
	2. Hardware/electrical interface issues between payload and spacecraft bus	2.9451276	4.457863799	2.924689458	4.58150592	3.148496586	4.757762118
	3. Payload malfunction due to mechanical issues	3.100044056	4.267186994	3.349764832	4.422110122	3.684628181	4.678684548
	4. Payload malfunction due to software issues	3.545703713	4.436069919	3.498190998	4.527144099	3.380372317	4.66821002

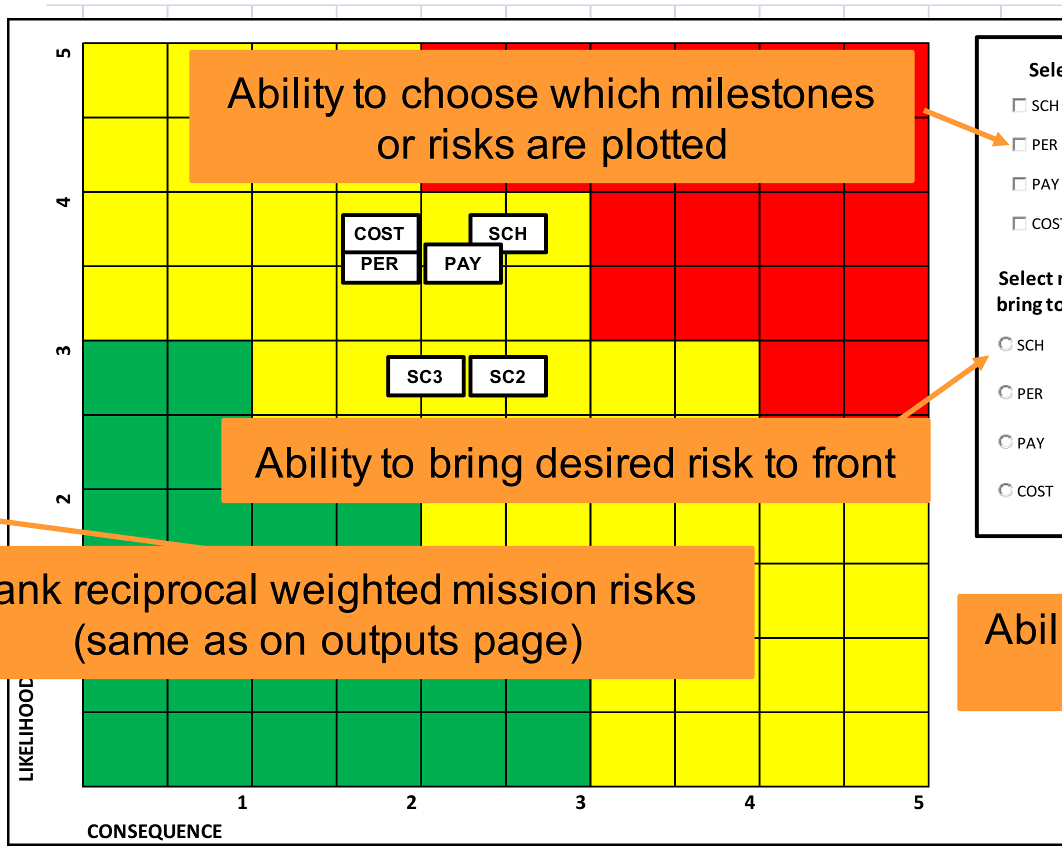
Currently up to 3 milestones can be tracked at one time

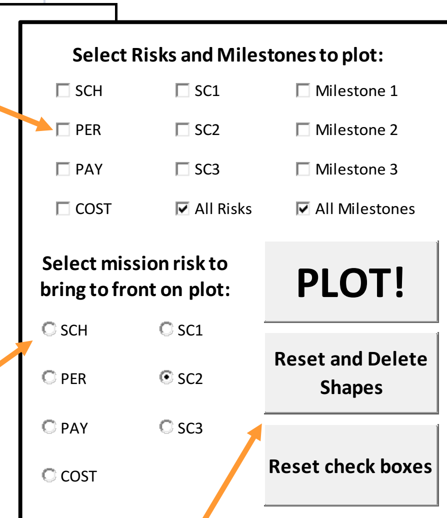
Mission risk L-C values calculated via rank reciprocal weighting scheme (see JoSS paper)

Root causes calculated via VBA-programmed functions

Risk Tool V1.1 – Plots Page

Milestone 1	C	L
SCH	2.44	3.9
PER	1.89	3.5
PAY	2.38	3.4
SC-1	2.42	2.79
SC-2	2.57	2.69
SC-3	2.07	2.88
COST	1.97	3.73





Ability to choose which milestones or risks are plotted

Ability to bring desired risk to front

Rank reciprocal weighted mission risks (same as on outputs page)

Ability to clear plot and reset options

Risk Tool V1.1 – Equations Pages

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Pages for both Consequence and Likelihood

Mission Risk	Root Cause	Consequence Formula	Consequence Formula #	Consequence coefficients													
				a	b	c	d	e	f	g	h	i	j	k	l	m	n
Schedule																	
	1. Inability to find desired spacecraft components	L3 = a + b * ff + cc * launch	3	3.854	-0.29	-0.25	0	0	0	0	0	0	0	0	0	0	0
	2. Mechanical design delays (such as issues with the CAD or drawings)	T2 = a + b*dev^cc + d*int^e + f*scfunc^g + h*environ^l + j*wait^k + l*ops^m	10	-231	1.68	-49.3	19.49	-0.01	159.4	3E-04	-0.01	-1	54.4	0.003	0.009	0.683	0
	3. Software design delays (such as basic component functionality or embedded coding issues)	L2 = a + b*ff + cc*dev + d*int + e*scfunc + f*environ + g*wait + h*ops	2	2.973	-0.02	-0	-0.02	-0.09	0.092	0.005	0.033	0	0	0	0	0	0
	4. Delay due to issuse with payload provider (may be related to delivery of EDU or flight unit, documentation, or interface issues)	L3 = a + b * ff + cc * launch	3	3.107	-0.04	0.194	0	0	0	0	0	0	0	0	0	0	0
	5. Delay due to inadequate documentation	T2 = a + b*dev^cc + d*int^e + f*scfunc^g + h*environ^l + j*wait^k + l*ops^m	10	-38.7	1.68	-49.3	1	0.12	0.025	0.023	-40.6	0.002	26.86	-0.01	55.3	-0.01	0
Payload																	
	1. Software interface issues between payload and spacecraft bus	L2 = a + b*ff + cc*dev + d*int + e*scfunc + f*environ + g*wait + h*ops	2	2.489	0.175	-0.01	-0.08	0.056	0.034	-0.01	0.008	0	0	0	0	0	0
	2. Hardware/electrical interface issues between payload and spacecraft bus	L2 = a + b*ff + cc*dev + d*int + e*scfunc + f*environ + g*wait + h*ops	2	3.312	-0.03	1E-04	-0.02	0.023	0.008	-0.06	3E-04	0	0	0	0	0	0
	3. Payload malfunction due to mechanical issues	L5 = a + b*dev + cc*int + d*scfunc + e*environ + f*wait + g*ops	5	3.501	-0	0.122	-0.02	-0.01	-0.03	-0.02	0	0	0	0	0	0	0
	4. Payload malfunction due to software issues	L2 = a + b*ff + cc*dev + d*int + e*scfunc + f*environ + g*wait + h*ops	2	3.194	0.113	-0	-0.03	-0.01	-0.05	-0.02	0.034	0	0	0	0	0	0

Formulas and coefficients as reference

Risk Tool – Obtain & Feedback

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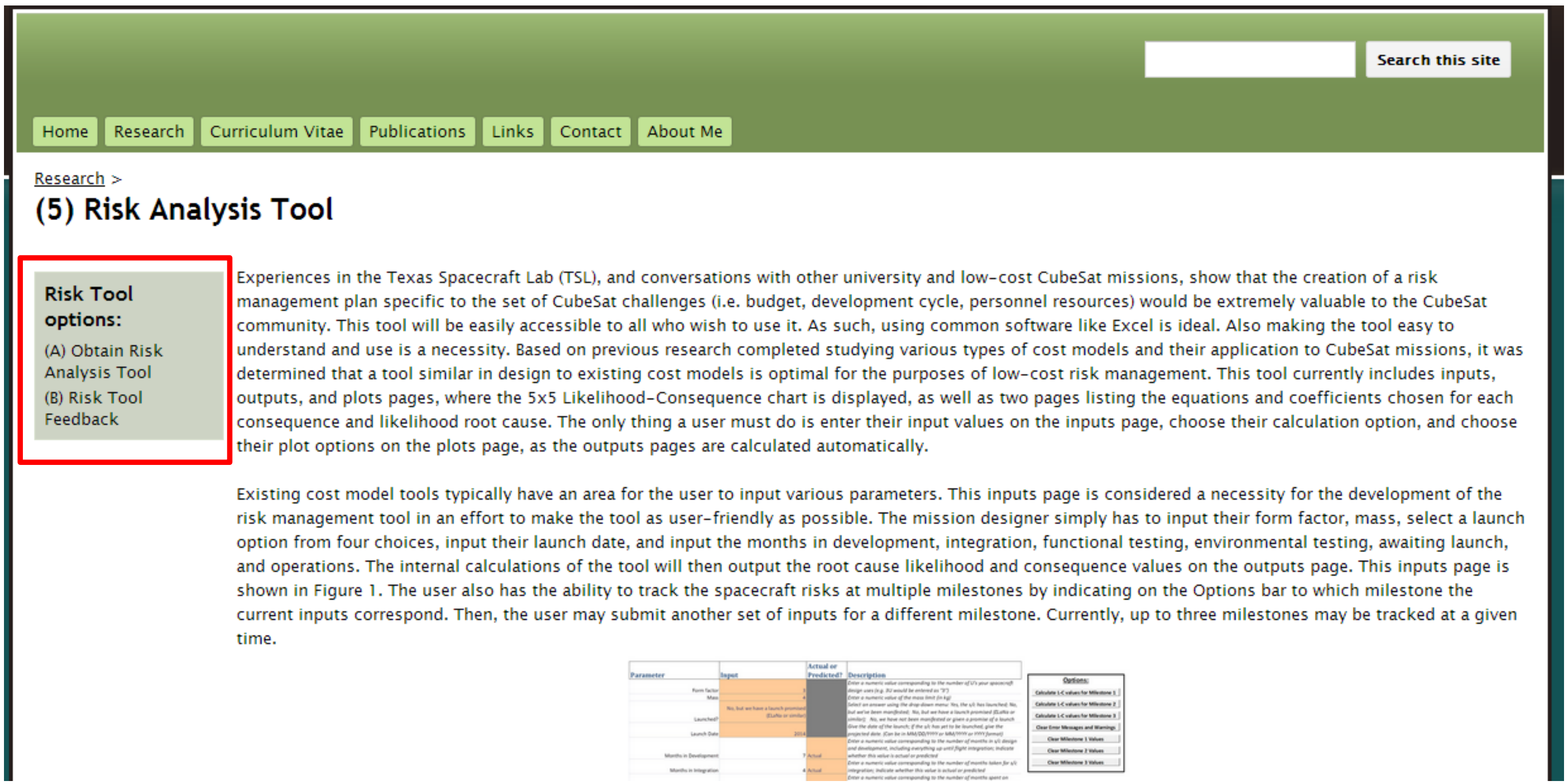
To obtain a **FREE** copy of the tool **AND** leave feedback on how to improve it:

https://sites.google.com/site/brumbaughresearch/research/risk-analysis-tool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Research >

(5) Risk Analysis Tool

Risk Tool options:

(A) Obtain Risk Analysis Tool

(B) Risk Tool Feedback

Experiences in the Texas Spacecraft Lab (TSL), and conversations with other university and low-cost CubeSat missions, show that the creation of a risk management plan specific to the set of CubeSat challenges (i.e. budget, development cycle, personnel resources) would be extremely valuable to the CubeSat community. This tool will be easily accessible to all who wish to use it. As such, using common software like Excel is ideal. Also making the tool easy to understand and use is a necessity. Based on previous research completed studying various types of cost models and their application to CubeSat missions, it was determined that a tool similar in design to existing cost models is optimal for the purposes of low-cost risk management. This tool currently includes inputs, outputs, and plots pages, where the 5x5 Likelihood-Consequence chart is displayed, as well as two pages listing the equations and coefficients chosen for each consequence and likelihood root cause. The only thing a user must do is enter their input values on the inputs page, choose their calculation option, and choose their plot options on the plots page, as the outputs pages are calculated automatically.

Existing cost model tools typically have an area for the user to input various parameters. This inputs page is considered a necessity for the development of the risk management tool in an effort to make the tool as user-friendly as possible. The mission designer simply has to input their form factor, mass, select a launch option from four choices, input their launch date, and input the months in development, integration, functional testing, environmental testing, awaiting launch, and operations. The internal calculations of the tool will then output the root cause likelihood and consequence values on the outputs page. This inputs page is shown in Figure 1. The user also has the ability to track the spacecraft risks at multiple milestones by indicating on the Options bar to which milestone the current inputs correspond. Then, the user may submit another set of inputs for a different milestone. Currently, up to three milestones may be tracked at a given time.

http://goo.gl/8hpPiz

Risk Tool – Obtain & Feedback

Texas Spacecraft Lab

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Research > (5) Risk Analysis Tool >

(A) Obtain Risk Analysis Tool

Risk Tool Options

(A) Obtain Risk Analysis Tool

(B) Risk Tool Feedback

To obtain a copy of the Risk Analysis Tool, please fill out the following form. You will then be contacted with a copy of the tool. For a description of how to use the tool as well as a discussion of the mathematical methods behind the tool, please read the User's Guide.

Request for CubeSat Risk Tool

In order to keep track of user demographics, please fill out the following information. Once the response is received, I will contact you with instructions on how to access the tool.

* Required

What is your name? *

What is your email address? *

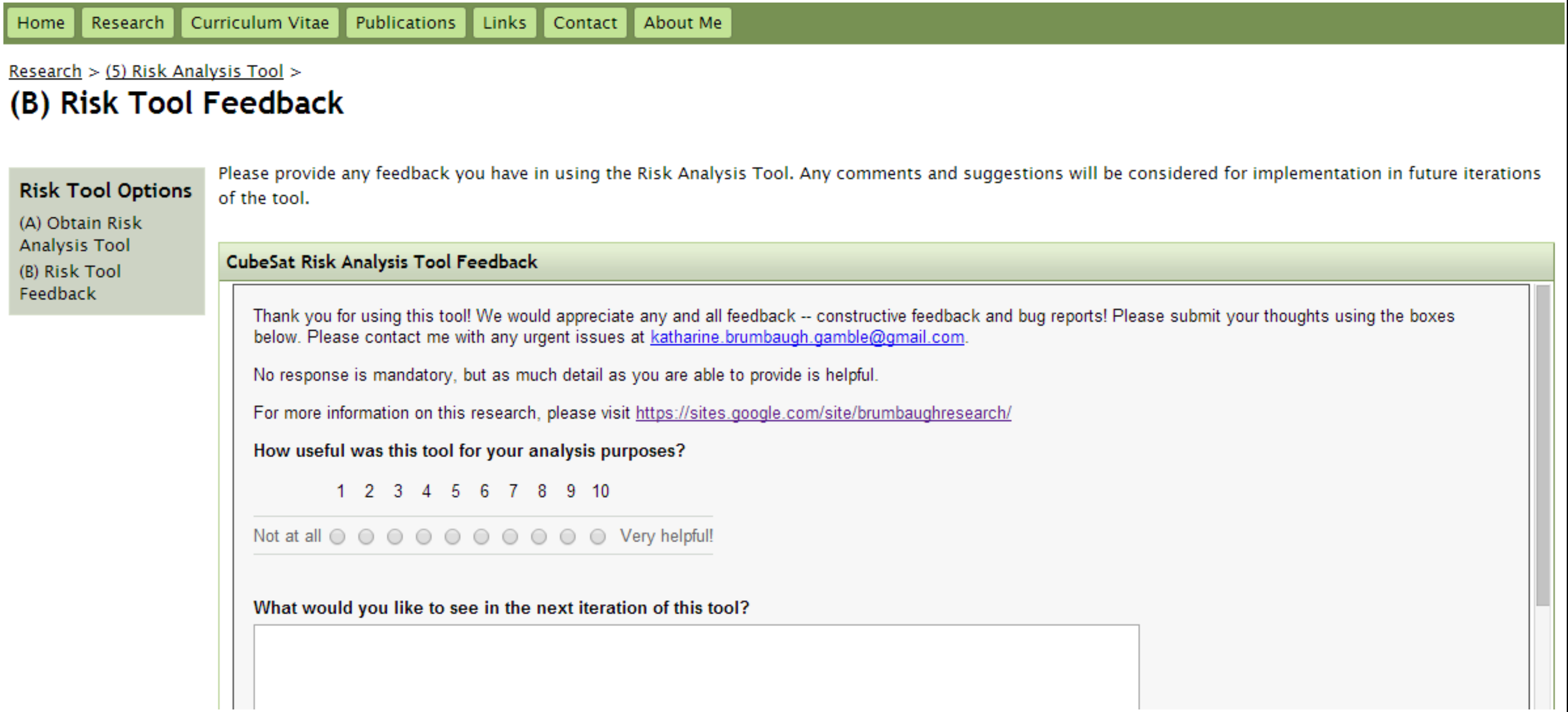
What organization are you with? *

University, company, etc.

What is the name of your CubeSat mission?

http://goo.gl/8hpPiz

Risk Tool – Obtain & Feedback



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Research > (5) Risk Analysis Tool >

(B) Risk Tool Feedback

Risk Tool Options

(A) Obtain Risk Analysis Tool

(B) Risk Tool Feedback

Please provide any feedback you have in using the Risk Analysis Tool. Any comments and suggestions will be considered for implementation in future iterations of the tool.

CubeSat Risk Analysis Tool Feedback

Thank you for using this tool! We would appreciate any and all feedback -- constructive feedback and bug reports! Please submit your thoughts using the boxes below. Please contact me with any urgent issues at katharine.brumbaugh.gamble@gmail.com.

No response is mandatory, but as much detail as you are able to provide is helpful.

For more information on this research, please visit https://sites.google.com/site/brumbaughresearch/

How useful was this tool for your analysis purposes?

	1	2	3	4	5	6	7	8	9	10	
Not at all	○	○	○	○	○	○	○	○	○	○	Very helpful!

What would you like to see in the next iteration of this tool?

http://goo.gl/8hpPiz

Decision Advisor – First Look

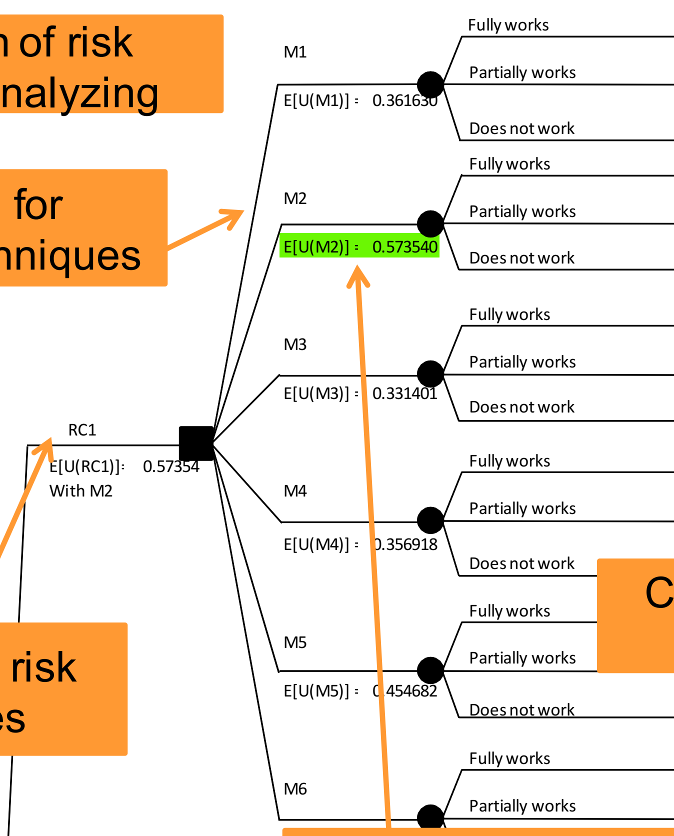
Macro to calculate everything

User defined utility values combined into multi-attribute joint utility function

Schedule risk - the event of a slip in meeting schedule milestones or deadlines that could be caused by any number of things. The five root causes identified here were used in the gathering of survey data.

Definition of risk currently analyzing

Find Max Utility



Branch probability	Implementation Cost ($ hundreds)	People Needed (#)	Time Estimate (days)	U(total)
p(F|M1,RC1) = 0.94	10.50742149	2	25.44229507	0.360388
p(P|M1,RC1) = 0.003	9.516868591	1	16.82756424	0.568043
p(D|M1,RC1) = 0.057	1.733045578	2	22.42229462	0.37087
p(F|M2,RC1) = 0.034	2.232103348	3	12.71202087	0.254969
p(P|M2,RC1) = 0.828	8.346748352	1	13.36458206	0.584367
p(D|M2,RC1) = 0.138	10.34435272	1	12.63389587	0.587015
p(F|M3,RC1) = 0.318	3.930301666	4	24.84733582	0.107882
p(P|M3,RC1) = 0.092	8.673877716	1	23.21655273	0.550904
p(D|M3,RC1) = 0.59	5.172748566	2	9.864292145	0.417727
p(F|M4,RC1) = 0.836	1.807489395	2	15.88572502	0.388842
p(P|M4,RC1) = 0.091	10.20341873	5	15.20442009	0.037153
[illegible]	[illegible]	[illegible]	[illegible]	…39095
[illegible]	[illegible]	[illegible]	[illegible]	…09107
[illegible]	[illegible]	[illegible]	[illegible]	…21429
p(D|M5,RC1) = 0.733	2.437047958	1	22.24947929	0.557146
p(F|M6,RC1) = 0.985	12.51644135	2	24.37513351	0.360781
p(P|M6,RC1) = 0.01	11.20952606	3	18.77529621	0.228236

Sub-trees for mitigation techniques

Sub-trees for risk root causes

Cost, Personnel, and Time are basis for the utility curves and values

Excel highlights the technique which maximizes the utility function and places the results under the RC branch

Final Thoughts…

Texas Spacecraft Lab

- Thank you for submitting your risk data!
- Check out research website
  - Publications list
  - More survey data analysis
  - More descriptions and details of research (risk analysis and decision advisor tools)
- **The risk analysis tool was designed for your use – PLEASE use it, and let me know how it works!**

QR code for obtaining Risk Tool (and Research website)

http://goo.gl/8hpPiz

https://sites.google.com/site/brumbaughresearch/research/risk-analysis-tool

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Back-Up Slides

Survey Analysis Results

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Summarized additional SCH issues:

- Delayed launch: manifest change, launch vehicle and primary payload schedule slip
- Student workforce issues: time commitment and turnover rate
- Electronics design delay
- Funding delays: sequestration, grants, internal funding fluctuations
- Payload development delay: when internal and external
- Change in providers
- Contract delays
- Re-scoping / de-scoping mission goals
- Environmental/Spacecraft testing delay
- Political disagreement: quality assurance measures, funding, poor management, and resistance of program

Summarized additional PAY issues:

- New technology unknowns
- Environmental concerns: test early and often to avoid late-stage mitigation (EMI/EMC)
- To buy or develop a payload was more expensive than anticipated
- Legal issues surrounding licensing
- Software desgin and testing: suggest using interface emulators

Summarized additional SC issues:

- Lack of proper requirements and testing at the subsystem level
- Unknown loss of contact

Summarized additional PER issues:

- Lack of resources: small team size, poor distribution of resources
- Lack of professionalism: students treating project as partial credit, poor documentation
- Distributed locations of personnel yields communication and management issues
- Loss of information and pace when lose team members
- Lack of institution support and resources
- Many people involved implies lots of management overhead
- ITAR regulations

Summarized additional COST issues:

- Poor/last minute travel planning
- Poor initial cost planning
- Cost of professional engineer reviews
- Needed additional equipment after unforseen hardware issues
- Additional hardware iterations
- Changing requirements/contractors midway through design
- Finding and obtaining funding
- Unexpected increase in payload/contracted development costs

Survey Analysis Results

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Summarized personnel requirements to be on the team:

- Application and interview process; Peer review process; peer recruitment
- Students paid, get school credit, or volunteer; Time commitment requirement
- US citizenship required; ITAR compliance
- Minimum GPA requirement
- Students keep logs of their progress
- Class standing requirement; full-time student
- Industry - relevant experience required; degree requirements
- Industry - selected by open competitions

General Suggestions:

- Biggest CubeSat issue is managing personnel change
- Push for early development on a fully built, flight-like, engineering model -- this allows for faster integration and testing of flight units

Regression Approach

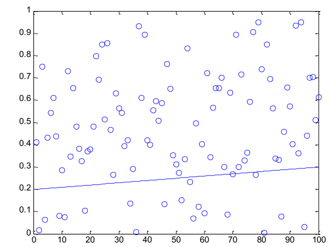
Input:
Variables of interest:
•Form factor
•Life cycle times
•Launch indicator

Calculation:
Regression techniques to minimize desired element(s):
•Sum of squared deviations
•Sum of error of the estimate
•Bias

$$SSD_M{}^2 = \sum_{i=1}^{n} \left(\frac{y_i - f(x_i, \underline{a})}{f(x_i, \underline{a})} \right)^2$$

$$B_M = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{f(x_i, \underline{a}) - y_i}{f(x_i, \underline{a})} \right)$$

Output:
Relationship between input variables and risk likelihood and consequence values: Y(X) = a*x_1^b + c*x_2^d + ...



Additive vs. Multiplicative Models

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	Additive	Multiplicative
Function form	$y_i = f(x_i, \vec{a}) + \varepsilon_i$	$y_i = f(x_i, \vec{a})\varepsilon_i$
Sum of Squared Deviations (SSD)	$SSD_A{}^2 = \sum_{i=1}^{n} \left(y_i - f(x_i, \underline{a})\right)^2$	$SSD_M{}^2 = \sum_{i=1}^{n} \left(\frac{y_i - f(x_i, \underline{a})}{f(x_i, \underline{a})} \right)^2$
Standard Error of Estimate (SEE)	$SEE_A = \sqrt{\frac{1}{n-m} \sum_{i=1}^{n} \left(y_i - f(x_i, \underline{a})\right)^2}$	$SEE_M = \sqrt{\frac{1}{n-m} \sum_{i=1}^{n} \left(\frac{y_i - f(x_i, \underline{a})}{f(x_i, \underline{a})} \right)^2}$
Bias	$B_A = \frac{1}{n} \sum_{i=1}^{n} \left(f(x_i, \underline{a}) - y_i \right)$	$B_M = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{f(x_i, \underline{a}) - y_i}{f(x_i, \underline{a})} \right)$
	Tends to favor larger values because of larger errors	Reduces influence of large data values

Use of General Error Regression (GER) will allow for use of either additive or multiplicative error models. Additionally, all function forms are available, as opposed to OLS methods where really only linear functions may be used.

Background: Regression Analysis

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- Given a set of data, regression analysis finds the line of best fit to describe the data
- Regression techniques include:
  - Ordinary Least Squares
    - Traditionally used for linear models and additive models
    - Minimizes square standard error
  - General Error Regression Techniques:
    - Can use additive or multiplicative functions.
    - Minimum Percentage Error (MPE)
    - Iterated Least Squares / Minimum Unbiased Percentage Error (IRLS / MUPE)
    - Minimum Percentage Error – Zero Percentage Bias (MPE-ZPE)

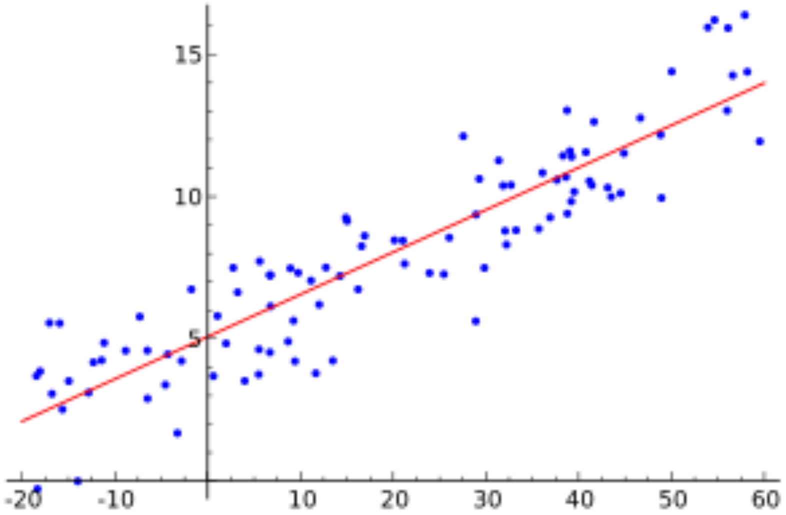

Image credit: Wikipedia

THE UNIVERSITY OF TEXAS AT AUSTIN
Cockrell School
Aerospace Engineering and Engineering Mechanics

"Lottery" System to Obtain U-Values

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What is the maximum amount of money you are willing to spend?	$ 2,500.00
What is the maximum number of people you are willing to allocate to a mitigation technique?	5
What is the maximum number of days you are able to spend on a mitigation technique?	40

If you had a 50-50 chance at losing $0 or losing $2500, what monetary value would you say is equivalent? Choose a value from the 'Value Outcomes table' and place it underneath the 'X'.

X ≈ $1,000.00

Probability		Outcome
0.5	A	$0.00
0.5	K	$2,500.00

If you had a 50-50 chance at losing $0 or losing $1000, what monetary value would you say is equivalent? Choose a value from the 'Value Outcomes table' and place it underneath the 'Y'.

Y ≈ $500.00

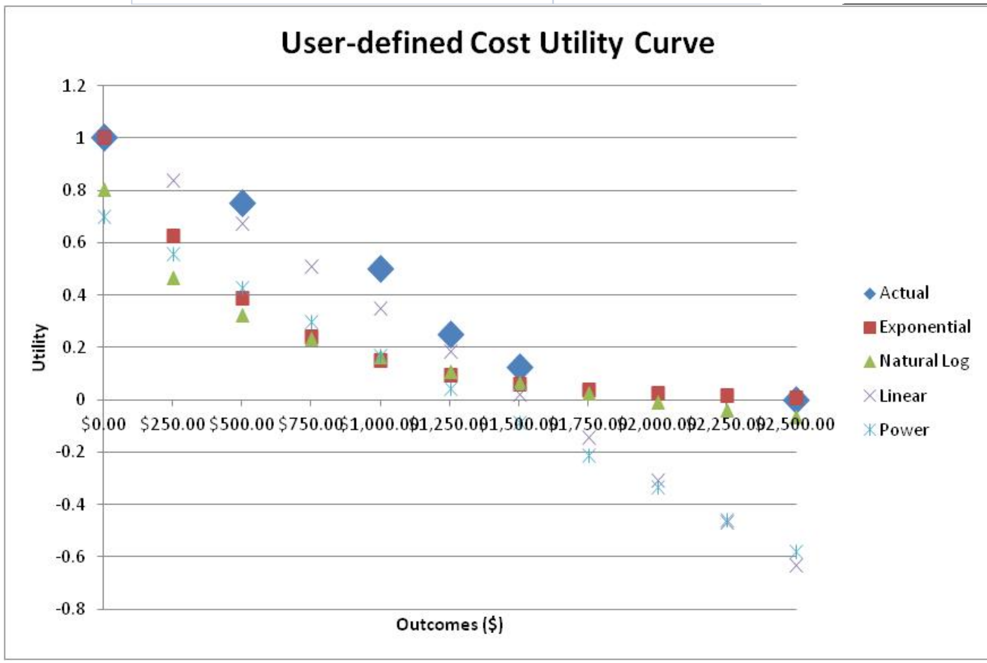
Probability		Outcome
0.5	A	$0.00
0.5	X	$1,000.00

...hance at losing $1000 ...hat monetary value ...equivalent? Choose a ...alue Outcomes table' ...nderneath the 'Z'.

Z ≈ $1,250.00

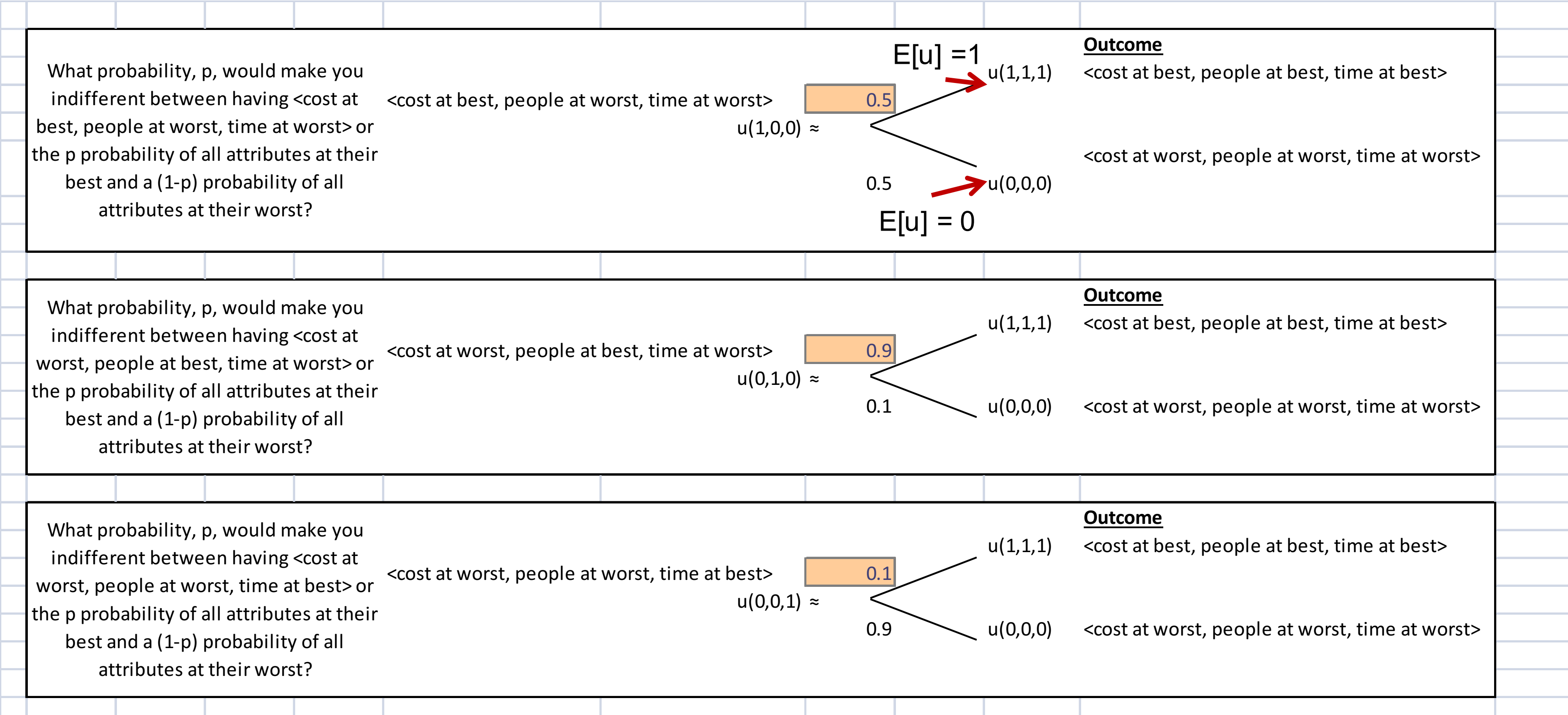
Probability		Outcome
0.5	A	$1,000.00
0.5	Y	$2,500.00

Value outcomes	
A	$0.00
B	$250.00
C	$500.00
D	$750.00
E	$1,000.00
F	$1,250.00
G	$1,500.00
H	$1,750.00
I	$2,000.00
J	$2,250.00
K	$2,500.00



"Lotteries" are used to obtain user preferences. Using several different lottery systems will ensure consistent results.

Joint Utility Curve – Scaling Factors



- Scaling constants must satisfy: $1 + k = (1 + kk_1)(1 + kk_2)(1 + kk_3)$
- k is determined by implicitly solving the equation given the k_i values obtained from the elicitation method

Joint Utility Function

- Once scaling constants have been found, may combine marginal utility functions with:

$$1 + ku(x_1, x_2, x_3) = (1 + k_1 u_1(x_1))(1 + k_2 u_2(x_2))(1 + k_3 u_3(x_3))$$

- Rescale the values to be between 0 and 1, with:
  - U(cost = min, people = min, time = min) = 1 → U(1,1,1)
  - U(cost = max, people = max, time = max) = 0 → U(0,0,0)

$$u' = \frac{u - u(0,0,0)}{u(1,1,1) - u(0,0,0)}$$

- Assumptions:
  - Three attributes fully characterize decision maker's preference system: cost, people, time required for a given mitigation technique
  - Preferential independence → tradeoffs between any two attributes governed by unique indifference relationship independent of other attribute
  - Utility independence → $u_i(x_i)$ is independent of all other $x_{j \neq i}$

Joint Utility Function

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- Example joint function w/ people = 3 :