A Software Tool for CubeSat Mission Risk Estimating Relationships

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- Labs located on campus in WRW building, 4th floor
- Entirely student-led with a faculty PI (Dr. Glenn Lightsey)
- Current flight experience:
  - FASTRAC nanosatellite (25 kg each), still operational, launched in Nov. 2010
  - Bevo-1/Paradigm (1U) launched in July 2009
- On the horizon:
  - RACE (3U) w/ JPL
    - Delivered spacecraft bus for radiometer mission, Mar 2014
    - To be flown via NanoRacks, October 2014
  - Bevo-2 (3U) w/ NASA-JSC & Texas A&M
    - Delivery to NASA in June 2014
    - To be flown via NanoRacks
  - ARMADILLO (3U) w/ Baylor University
    - University Nanosatellite Program winner, Jan. 2013
    - Selected for ELaNa in Spring 2012 (to be manifested)
  - INSPIRE (3U) w/ JPL
    - Providing thruster; collaboration with other organizations
    - To be flown on interplanetary trajectory
Talk Overview

• CubeSat Mission Risk Survey – Results!

• Risk Analysis Tool
  – Mathematical overview
  – Tool overview
  – How to obtain a copy (Feedback wanted!)

• Decision Advisor

Six main survey areas:
1. Demographics
2. Schedule Risk
3. Payload Risk
4. Spacecraft Risk – comm, basic health data, standards
5. Personnel & Management Risk
6. Cost Risk
Survey Data Results

- **THANK YOU!**
- Collected responses April – November 2013
- 65 CubeSat responses
- 52 unique and valid missions used for high-level analysis
- 3 outliers were removed for regression analysis
Survey Data Results

Years of Experience:
- 34 in (1.5)
- 7 in (0.1)
- 5 in (5.10)
- 3 in (3.15)
- 4 in (4.20)
- 6 in >20

Organization Responsible:
- Universities only: 31 (48%)
- Companies only: 7 (11%)
- Government only: 7 (11%)
- Univ partnerships: 1 (2%)
- Univ & Corp partnerships: 1 (2%)
- Univ & Gov partnerships: 13 (20%)
- Company & Gov partnerships: 2 (3%)
- Conglomerate: 2 (3%)

Type of Funding:
- Internal Funding: 44
- Non-Competitive award: 18
- Competitive award: 55
- Grant & Sponsorship: 6
Top Ten Risk Events as identified by survey responses:

1. Software design delay
2. Attrition or turnover of team members
3. Mechanical design delay
4. Incomplete understanding of the projected total mission cost
5. Inability to find desired spacecraft components
6. Sudden loss of crucial team members
7. Delay due to inadequate documentation
8. Loss of information
9. Lack of sufficient training for team members completing flight qualification necessary tasks
10. Delay due to issues with payload provider

All events in the top ten have to do with personnel, schedule, or cost
Risk Tool – Mathematical Overview

- Used General Error Regression, Minimum Percentage Error – Zero Percentage Bias (MPE-ZPB)
  - Similar regression method used by USCM and SSCM cost models
  - Comparable to Least Squares, but multiplicative error
- 12 Function Forms tested
- Used Excel VBA and Solver
## Factors of interest in regression analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Input</th>
<th>Actual or Predicted?</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Form factor</td>
<td>3</td>
<td>Predicted</td>
<td>Enter a numeric value corresponding to the number of U's your spacecraft design uses (e.g. 3U would be entered as &quot;3&quot;)</td>
</tr>
<tr>
<td>Mass</td>
<td>4</td>
<td>Predicted</td>
<td>Enter a numeric value of the mass limit (in kg)</td>
</tr>
<tr>
<td>Launched?</td>
<td></td>
<td></td>
<td>Select an answer using the drop-down menu: Yes, the s/c has launched; No, but we've been manifested; No, but we have a launch promised (ELaNa or similar); No, we have not been manifested or given a promise of a launch</td>
</tr>
<tr>
<td>Launch Date</td>
<td>2014</td>
<td></td>
<td>Give the date of the launch; if the s/c has yet to be launched, give the projected date. (Can be in MM/DD/YYYY or MM/YYYY or YYYY format)</td>
</tr>
<tr>
<td>Months in Development</td>
<td>7</td>
<td>Actual</td>
<td>Enter a numeric value corresponding to the number of months in s/c design and development, including everything up until flight integration; Indicate whether this value is actual or predicted</td>
</tr>
<tr>
<td>Months in Integration</td>
<td>4</td>
<td>Actual</td>
<td>Enter a numeric value corresponding to the number of months taken for s/c integration; Indicate whether this value is actual or predicted</td>
</tr>
<tr>
<td>Months in S/C Functional Testing</td>
<td>5</td>
<td>Predicted</td>
<td>Enter a numeric value corresponding to the number of months spent on integrated s/c testing at the organization level, including functional testing; Indicate whether this value is actual or predicted</td>
</tr>
<tr>
<td>Months in S/C Environmental Testing</td>
<td>3</td>
<td>Predicted</td>
<td>Enter a numeric value corresponding to the number of months spent on necessary testing to satisfy launch, thermal vac, vib tables, and mass value is actual or predicted</td>
</tr>
<tr>
<td>Months S/C is awaiting launch</td>
<td>3</td>
<td>Predicted</td>
<td>Enter a numeric value corresponding to the number of months the spacecraft was &quot;on the shelf&quot; waiting to be completed; Indicate whether this value is actual or predicted</td>
</tr>
<tr>
<td>Months S/C is in operations</td>
<td>6</td>
<td>Predicted</td>
<td>Enter a numeric value corresponding to the number of months the spacecraft was operational in orbit; Indicate whether this value is actual or predicted</td>
</tr>
<tr>
<td>Milestone</td>
<td>LVINT</td>
<td></td>
<td>Enter the name of the milestone for which these numbers reflect the status</td>
</tr>
</tbody>
</table>

### Options:
- Calculate L-C values for Milestone 1
- Calculate L-C values for Milestone 2
- Calculate L-C values for Milestone 3
- Clear Error Messages and Warnings
- Clear Milestone 1 Values
- Clear Milestone 2 Values
- Clear Milestone 3 Values

Macro buttons will calculate the L-C values for multiple milestones.

Indicate whether values are actual or predicted.
## Risk Tool V1.1 – Outputs Page

<table>
<thead>
<tr>
<th>Mission Risk</th>
<th>Root Cause</th>
<th>Consequence value</th>
<th>Likelihood value</th>
<th>Consequence value</th>
<th>Likelihood value</th>
<th>Consequence value</th>
<th>Likelihood value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schedule</td>
<td>1. Inability to find desired spacecraft components</td>
<td>3.31804782</td>
<td>4.487230123</td>
<td>3.290284849</td>
<td>4.471377851</td>
<td>3.1773186</td>
<td>4.535742296</td>
</tr>
<tr>
<td></td>
<td>2. Mechanical design delays (such as issues with the CAD or drawings)</td>
<td>2.212231943</td>
<td>4.352609117</td>
<td>2.212231943</td>
<td>4.392936827</td>
<td>2.212231943</td>
<td>4.557244851</td>
</tr>
<tr>
<td></td>
<td>3. Software design delays (such as basic component functionality or embedded coding issues)</td>
<td>3.941404598</td>
<td>4.774450701</td>
<td>3.869117987</td>
<td>4.699644896</td>
<td>3.431920442</td>
<td>4.693574931</td>
</tr>
<tr>
<td></td>
<td>4. Delay due to issue with payload provider (may be related to delivery of the EDU or flight unit, documentation, or interface issues)</td>
<td>6.24310987</td>
<td>4.293131741</td>
<td>6.564926097</td>
<td>4.405439749</td>
<td>4.13069613</td>
<td>4.853501468</td>
</tr>
<tr>
<td></td>
<td>5. Delay due to inadequate documentation</td>
<td>2.202138245</td>
<td>3.960906613</td>
<td>2.375918631</td>
<td>4.055460527</td>
<td>3.130304667</td>
<td>4.242332588</td>
</tr>
<tr>
<td>Payload</td>
<td>1. Software interface issues between payload and spacecraft bus</td>
<td>3.319286913</td>
<td>4.716094717</td>
<td>3.021747664</td>
<td>4.713521567</td>
<td>2.931091293</td>
<td>4.853501468</td>
</tr>
<tr>
<td></td>
<td>2. Hardware/electrical interface issues between payload and spacecraft bus</td>
<td>2.9451276</td>
<td>4.457863799</td>
<td>2.92469458</td>
<td>4.58150592</td>
<td>3.148496586</td>
<td>4.757762118</td>
</tr>
</tbody>
</table>

Currently up to 3 milestones can be tracked at one time

Mission risk L-C values calculated via rank reciprocal weighting scheme (see JoSS paper)

Root causes calculated via VBA-programmed functions
Risk Tool V1.1 – Plots Page

Ability to choose which milestones or risks are plotted

Ability to bring desired risk to front

Rank reciprocal weighted mission risks (same as on outputs page)

Ability to clear plot and reset options
## Risk Tool V1.1 – Equations Pages

The table below shows the risk equations and coefficients for both consequence and likelihood.

### Mission Risk

<table>
<thead>
<tr>
<th>Schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Inability to find desired spacecraft components</td>
</tr>
<tr>
<td>2. Mechanical design delays (such as issues with the CAD or drawings)</td>
</tr>
<tr>
<td>3. Software design delays (such as basic component functionality or embedded coding issues)</td>
</tr>
<tr>
<td>4. Delay due to issue with payload provider (may be related to delivery of EDU or flight unit, documentation, or interface issues)</td>
</tr>
<tr>
<td>5. Delay due to inadequate documentation</td>
</tr>
</tbody>
</table>

### Payload

<table>
<thead>
<tr>
<th>Payload</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Software interface issues between payload and spacecraft bus</td>
</tr>
<tr>
<td>2. Hardware/electrical interface issues between payload and spacecraft bus</td>
</tr>
<tr>
<td>3. Payload malfunction due to mechanical issues</td>
</tr>
<tr>
<td>4. Payload malfunction due to software issues</td>
</tr>
</tbody>
</table>

### Consequence Formula

- **Schedule**
  - \( L_3 = a + b \cdot ff + cc \cdot launch \)
  - \( T_2 = a + b \cdot dev \cdot cc + d \cdot int \cdot e + f \cdot scfunc \cdot g + h \cdot environ \cdot i + j \cdot wait \cdot k + l \cdot ops \cdot m \)
  - \( L_2 = a + b \cdot ff + cc \cdot dev + d \cdot int + e \cdot scfunc + f \cdot environ + g \cdot wait + h \cdot ops \)
  - \( L_5 = a + b \cdot dev + cc \cdot int + d \cdot scfunc + e \cdot environ + f \cdot wait + g \cdot ops \)
  - \( L_2 = a + b \cdot ff + cc \cdot dev + d \cdot int + e \cdot scfunc + f \cdot environ + g \cdot wait + h \cdot ops \)

### Consequence Formulas and Coefficients

<table>
<thead>
<tr>
<th>Schedule</th>
<th>Coefficient</th>
<th>Consequence Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Inability to find desired spacecraft components</td>
<td>( L_3 = a + b \cdot ff + cc \cdot launch )</td>
<td>3.854</td>
</tr>
<tr>
<td>2. Mechanical design delays (such as issues with the CAD or drawings)</td>
<td>( T_2 = a + b \cdot dev \cdot cc + d \cdot int \cdot e + f \cdot scfunc \cdot g + h \cdot environ \cdot i + j \cdot wait \cdot k + l \cdot ops \cdot m )</td>
<td>-231</td>
</tr>
<tr>
<td>3. Software design delays (such as basic component functionality or embedded coding issues)</td>
<td>( L_2 = a + b \cdot ff + cc \cdot dev + d \cdot int + e \cdot scfunc + f \cdot environ + g \cdot wait + h \cdot ops )</td>
<td>2.973</td>
</tr>
<tr>
<td>4. Delay due to issue with payload provider (may be related to delivery of EDU or flight unit, documentation, or interface issues)</td>
<td>( L_3 = a + b \cdot ff + cc \cdot launch )</td>
<td>3.107</td>
</tr>
<tr>
<td>5. Delay due to inadequate documentation</td>
<td>( T_2 = a + b \cdot dev \cdot cc + d \cdot int \cdot e + f \cdot scfunc \cdot g + h \cdot environ \cdot i + j \cdot wait \cdot k + l \cdot ops \cdot m )</td>
<td>-38.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Payload</th>
<th>Coefficient</th>
<th>Consequence Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Software interface issues between payload and spacecraft bus</td>
<td>( L_2 = a + b \cdot ff + cc \cdot dev + d \cdot int + e \cdot scfunc + f \cdot environ + g \cdot wait + h \cdot ops )</td>
<td>2.489</td>
</tr>
<tr>
<td>2. Hardware/electrical interface issues between payload and spacecraft bus</td>
<td>( L_2 = a + b \cdot ff + cc \cdot dev + d \cdot int + e \cdot scfunc + f \cdot environ + g \cdot wait + h \cdot ops )</td>
<td>2.312</td>
</tr>
<tr>
<td>3. Payload malfunction due to mechanical issues</td>
<td>( L_5 = a + b \cdot dev + cc \cdot int + d \cdot scfunc + e \cdot environ + f \cdot wait + g \cdot ops )</td>
<td>3.501</td>
</tr>
<tr>
<td>4. Payload malfunction due to software issues</td>
<td>( L_2 = a + b \cdot ff + cc \cdot dev + d \cdot int + e \cdot scfunc + f \cdot environ + g \cdot wait + h \cdot ops )</td>
<td>3.194</td>
</tr>
</tbody>
</table>
To obtain a **FREE** copy of the tool **AND** leave feedback on how to improve it:

https://sites.google.com/site/brumbaughresearch/research/risk-analysis-tool

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**Risk Tool options:**

(a) Obtain Risk Analysis Tool
(b) Risk Tool Feedback

Experiences in the Texas Spacecraft Lab (TSL), and conversations with other university and low-cost CubeSat missions, show that the creation of a risk management plan specific to the set of CubeSat challenges (i.e. budget, development cycle, personnel resources) would be extremely valuable to the CubeSat community. This tool will be easily accessible to all who wish to use it. As such, using common software like Excel is ideal. Also making the tool easy to understand and use is a necessity. Based on previous research completed studying various types of cost models and their application to CubeSat missions, it was determined that a tool similar in design to existing cost models is optimal for the purposes of low-cost risk management. This tool currently includes inputs, outputs, and plots pages, where the 5x5 Likelihood-Consequence chart is displayed, as well as two pages listing the equations and coefficients chosen for each consequence and likelihood root cause. The only thing a user must do is enter their input values on the inputs page, choose their calculation option, and choose their plot options on the plots page, as the outputs pages are calculated automatically.

Existing cost model tools typically have an area for the user to input various parameters. This inputs page is considered a necessity for the development of the risk management tool in an effort to make the tool as user-friendly as possible. The mission designer simply has to input their form factor, mass, select a launch option from four choices, input their launch date, and input the months in development, integration, functional testing, environmental testing, awaiting launch, and operations. The internal calculations of the tool will then output the root cause likelihood and consequence values on the outputs page. This inputs page is shown in Figure 1. The user also has the ability to track the spacecraft risks at multiple milestones by indicating on the Options bar to which milestone the current inputs correspond. Then, the user may submit another set of inputs for a different milestone. Currently, up to three milestones may be tracked at a given time.

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http://goo.gl/8hpPiz
(A) Obtain Risk Analysis Tool

To obtain a copy of the Risk Analysis Tool, please fill out the following form. You will then be contacted with a copy of the tool. For a description of how to use the tool as well as a discussion of the mathematical methods behind the tool, please read the User's Guide.

Request for CubeSat Risk Tool

In order to keep track of user demographics, please fill out the following information. Once the response is received, I will contact you with instructions on how to access the tool.

* Required

What is your name? *

What is your email address? *

What organization are you with? *

University, company, etc.

What is the name of your CubeSat mission?

http://goo.gl/8hpPiz
(B) Risk Tool Feedback

Please provide any feedback you have in using the Risk Analysis Tool. Any comments and suggestions will be considered for implementation in future iterations of the tool.

CubeSat Risk Analysis Tool Feedback

Thank you for using this tool! We would appreciate any and all feedback – constructive feedback and bug reports! Please submit your thoughts using the boxes below. Please contact me with any urgent issues at katherine.brumbaugh.gamble@gmail.com.

No response is mandatory, but as much detail as you are able to provide is helpful.

For more information on this research, please visit https://sites.google.com/site/brumbaughresearch/

How useful was this tool for your analysis purposes?

1 2 3 4 5 6 7 8 9 10

Not at all 🈴 🈴 🈴 🈴 🈴 🈴 🈴 🈴 Very helpful!

What would you like to see in the next iteration of this tool?

http://goo.gl/8hpPiz
### Schedule Risk

The event of a slip in meeting schedule milestones or deadlines that could be caused by any number of things. The five root causes identified here were used in the gathering of survey data.

### User defined utility values combined into multi-attribute joint utility function

<table>
<thead>
<tr>
<th>Implementation Cost ($ hundreds)</th>
<th>People Needed ( # )</th>
<th>Time Estimate (days)</th>
<th>U(\text{total})</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.50742249</td>
<td>2</td>
<td>25.44229507</td>
<td>0.360388</td>
</tr>
<tr>
<td>9.516868591</td>
<td>1</td>
<td>16.82756424</td>
<td>0.568043</td>
</tr>
<tr>
<td>1.733045578</td>
<td>2</td>
<td>22.42229462</td>
<td>0.37087</td>
</tr>
<tr>
<td>2.232103348</td>
<td>3</td>
<td>12.71202087</td>
<td>0.254969</td>
</tr>
<tr>
<td>8.346748352</td>
<td>1</td>
<td>13.36582606</td>
<td>0.584367</td>
</tr>
<tr>
<td>10.34435272</td>
<td>1</td>
<td>12.6389587</td>
<td>0.587015</td>
</tr>
<tr>
<td>3.930301666</td>
<td>4</td>
<td>24.8473582</td>
<td>0.107882</td>
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<tr>
<td>8.673877716</td>
<td>1</td>
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<td>0.55904</td>
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<tr>
<td>5.172748566</td>
<td>2</td>
<td>9.864292145</td>
<td>0.417727</td>
</tr>
<tr>
<td>1.807489395</td>
<td>2</td>
<td>15.8652502</td>
<td>0.38842</td>
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<tr>
<td>10.20341873</td>
<td>5</td>
<td>15.20442009</td>
<td>0.037153</td>
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<td>12.51644135</td>
<td>2</td>
<td>24.37513351</td>
<td>0.360781</td>
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<tr>
<td>11.20952606</td>
<td>3</td>
<td>18.77529621</td>
<td>0.228236</td>
</tr>
</tbody>
</table>

**Cost, Personnel, and Time are basis for the utility curves and values**

**Excel highlights the technique which maximizes the utility function and places the results under the RC branch**

**Macro to calculate everything**

**Definition of risk currently analyzing**

**Sub-trees for mitigation techniques**

**Sub-trees for risk root causes**

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**Substitution**

Macro to calculate everything

User defined utility values combined into multi-attribute joint utility function

Definition of risk currently analyzing

Sub-trees for mitigation techniques

Sub-trees for risk root causes

Excel highlights the technique which maximizes the utility function and places the results under the RC branch
Final Thoughts…

• Thank you for submitting your risk data!
• Check out research website
  – Publications list
  – More survey data analysis
  – More descriptions and details of research (risk analysis and decision advisor tools)
• The risk analysis tool was designed for your use – PLEASE use it, and let me know how it works!

https://sites.google.com/site/brumbaughresearch/research/risk-analysis-tool

QR code for obtaining Risk Tool (and Research website)
Back-Up Slides
### Survey Analysis Results

#### Summarized additional SCH issues:
- Delayed launch: manifest change, launch vehicle and primary payload schedule slip
- Student workforce issues: time commitment and turnover rate
- Electronics design delay
- Funding delays: sequestration, grants, internal funding fluctuations
- Payload development delay: when internal and external
- Change in providers
- Contract delays
- Re-scoping / de-scoping mission goals
- Environmental/Spacecraft testing delay
- Political disagreement: quality assurance measures, funding, poor management, and resistance of program

#### Summarized additional SC issues:
- Lack of proper requirements and testing at the subsystem level
- Unknown loss of contact

#### Summarized additional PER issues:
- Lack of resources: small team size, poor distribution of resources
- Lack of professionalism: students treating project as partial credit, poor documentation
- Distributed locations of personnel yields communication and management issues
- Loss of information and pace when lose team members
- Lack of institution support and resources
- Many people involved implies lots of management overhead
- ITAR regulations

#### Summarized additional PAY issues:
- New technology unknowns
- Environmental concerns: test early and often to avoid late-stage mitigation (EMI/EMC)
- To buy or develop a payload was more expensive than anticipated
- Legal issues surrounding licensing
- Software design and testing: suggest using interface emulators

#### Summarized additional COST issues:
- Poor/last minute travel planning
- Poor initial cost planning
- Cost of professional engineer reviews
- Needed additional equipment after unforeseen hardware issues
- Additional hardware iterations
- Changing requirements/contractors midway through design
- Finding and obtaining funding
- Unexpected increase in payload/contracted development costs
Summarized personnel requirements to be on the team:

• Application and interview process; Peer review process; peer recruitment
• Students paid, get school credit, or volunteer; Time commitment requirement
• US citizenship required; ITAR compliance
• Minimum GPA requirement
• Students keep logs of their progress
• Class standing requirement; full-time student
• Industry - relevant experience required; degree requirements
• Industry - selected by open competitions

General Suggestions:

• Biggest CubeSat issue is managing personnel change
• Push for early development on a fully built, flight-like, engineering model -- this allows for faster integration and testing of flight units
Regression Approach

**Input:**
Variables of interest:
- Form factor
- Life cycle times
- Launch indicator

**Calculation:**
Regression techniques to minimize desired element(s):
- Sum of squared deviations
- Sum of error of the estimate
- Bias

$$SSD_M = \sum_{i=1}^{n} \left( \frac{y_i - f(x_i, a)}{f(x_i, a)} \right)^2$$

$$B_M = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{f(x_i, a) - y_i}{f(x_i, a)} \right)$$

**Output:**
Relationship between input variables and risk likelihood and consequence values: $Y(X) = a \cdot x_1^b + c \cdot x_2^d + ...$
### Additive vs. Multiplicative Models

<table>
<thead>
<tr>
<th></th>
<th>Additive</th>
<th>Multiplicative</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Function form</strong></td>
<td>$y_i = f(x_i, \bar{a}) + \varepsilon_i$</td>
<td>$y_i = f(x_i, \bar{a})\varepsilon_i$</td>
</tr>
<tr>
<td><strong>Sum of Squared Deviations (SSD)</strong></td>
<td>$SSD_A^2 = \sum_{i=1}^{n} (y_i - f(x_i, a))^2$</td>
<td>$SSD_M^2 = \sum_{i=1}^{n} \left( \frac{y_i - f(x_i, a)}{f(x_i, a)} \right)^2$</td>
</tr>
<tr>
<td><strong>Standard Error of Estimate (SEE)</strong></td>
<td>$SEE_A = \sqrt{\frac{1}{n-m} \sum_{i=1}^{n} (y_i - f(x_i, a))^2}$</td>
<td>$SEE_M = \sqrt{\frac{1}{n-m} \sum_{i=1}^{n} \left( \frac{y_i - f(x_i, a)}{f(x_i, a)} \right)^2}$</td>
</tr>
<tr>
<td><strong>Bias</strong></td>
<td>$B_A = \frac{1}{n} \sum_{i=1}^{n} (f(x_i, a) - y_i)$</td>
<td>$B_M = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{f(x_i, a) - y_i}{f(x_i, a)} \right)$</td>
</tr>
</tbody>
</table>

- **Additive**: Tends to favor larger values because of larger errors.
- **Multiplicative**: Reduces influence of large data values.

Use of General Error Regression (GER) will allow for use of either additive or multiplicative error models. Additionally, all function forms are available, as opposed to OLS methods where really only linear functions may be used.
Background: Regression Analysis

- Given a set of data, regression analysis finds the line of best fit to describe the data.

- Regression techniques include:
  - Ordinary Least Squares
    - Traditionally used for linear models and additive models
    - Minimizes square standard error
  - General Error Regression Techniques:
    - Can use additive or multiplicative functions.
    - Minimum Percentage Error (MPE)
    - Iterated Least Squares / Minimum Unbiased Percentage Error (IRLS / MUPE)
    - Minimum Percentage Error – Zero Percentage Bias (MPE-ZPE)

“Lottery” System to Obtain U-Values

If you had a 50-50 chance at losing $0 or losing $2500, what monetary value would you say is equivalent? Choose a value from the ‘Value Outcomes table’ and place it underneath the ‘X’.

\[ X = 0.5 \]
\[ X \approx \frac{K}{2} \]
\[ X = $1000.00 \]
\[ X = 0.5 \]
\[ X = $1000.00 \]

If you had a 50-50 chance at losing $0 or losing $1000, what monetary value would you say is equivalent? Choose a value from the ‘Value Outcomes table’ and place it underneath the ‘Y’.

\[ Y = 0.5 \]
\[ Y = \frac{K}{2} \]
\[ Y = $500.00 \]
\[ Y = 0.5 \]
\[ Y = $500.00 \]

If you had a 50-50 chance at losing $1000 or losing $3000, what monetary value would you say is equivalent? Choose a value from the ‘Value Outcomes table’ and place it underneath the ‘Z’.

\[ Z = 0.5 \]
\[ Z = \frac{K}{2} \]
\[ Z = $1250.00 \]
\[ Z = 0.5 \]
\[ Z = $1250.00 \]

“Lotteries” are used to obtain user preferences. Using several different lottery systems will ensure consistent results.
### Joint Utility Curve – Scaling Factors

What probability, p, would make you indifferent between having <cost at best, people at worst, time at worst> or the p probability of all attributes at their best and a (1-p) probability of all attributes at their worst?

<table>
<thead>
<tr>
<th>Outcome</th>
<th>&lt;cost at best, people at worst, time at best&gt;</th>
<th>E[u] = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>u(1,1,1)</td>
<td>&lt;cost at worst, people at worst, time at worst&gt;</td>
<td>0.5</td>
</tr>
<tr>
<td>u(0,0,0)</td>
<td>&lt;cost at worst, people at worst, time at worst&gt;</td>
<td>0.5</td>
</tr>
<tr>
<td>E[u] = 0</td>
<td>&lt;cost at worst, people at worst, time at worst&gt;</td>
<td></td>
</tr>
</tbody>
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What probability, p, would make you indifferent between having <cost at worst, people at best, time at worst> or the p probability of all attributes at their best and a (1-p) probability of all attributes at their worst?

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<td>u(1,1,1)</td>
<td>&lt;cost at worst, people at worst, time at worst&gt;</td>
<td>0.9</td>
</tr>
<tr>
<td>u(0,1,0)</td>
<td>&lt;cost at worst, people at worst, time at worst&gt;</td>
<td>0.1</td>
</tr>
<tr>
<td>u(0,0,0)</td>
<td>&lt;cost at worst, people at worst, time at worst&gt;</td>
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What probability, p, would make you indifferent between having <cost at worst, people at worst, time at best> or the p probability of all attributes at their best and a (1-p) probability of all attributes at their worst?

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<td>0.9</td>
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<tr>
<td>E[u] = 0</td>
<td>&lt;cost at worst, people at worst, time at best&gt;</td>
<td></td>
</tr>
</tbody>
</table>

• Scaling constants must satisfy: \(1 + k = (1 + kk_1)(1 + kk_2)(1 + kk_3)\)
• \(k\) is determined by implicitly solving the equation given the \(k_i\) values obtained from the elicitation method
Joint Utility Function

- Once scaling constants have been found, may combine marginal utility functions with:

\[ 1 + ku(x_1, x_2, x_3) = (1 + k_1 u_1(x_1))(1 + k_2 u_2(x_2))(1 + k_3 u_3(x_3)) \]

- Rescale the values to be between 0 and 1, with:
  - \( U(\text{cost} = \text{min}, \text{people} = \text{min}, \text{time} = \text{min}) = 1 \rightarrow U(1,1,1) \)
  - \( U(\text{cost} = \text{max}, \text{people} = \text{max}, \text{time} = \text{max}) = 0 \rightarrow U(0,0,0) \)

\[ u' = \frac{u - u(0,0,0)}{u(1,1,1) - u(0,0,0)} \]

- Assumptions:
  - Three attributes fully characterize decision maker’s preference system: cost, people, time required for a given mitigation technique
  - Preferential independence \( \rightarrow \) tradeoffs between any two attributes governed by unique indifference relationship independent of other attribute
  - Utility independence \( \rightarrow \) \( u_i(x_i) \) is independent of all other \( x_{j \neq i} \)
Joint Utility Function

- Example joint function w/ people = 3: