THE UNIVERSITY OF TEXAS AT AUSTIN

WHAT STARTS HERE CHANGES THE WORLD
Stochastic Attitude Control of a CubeSat

Jason Moore
Advisor: Dr Robert H Bishop
UT Austin Satellite Program

• Lonestar
  – Joint NASA / UT Austin / Texas A&M
  – Multi-year program
  – Paradigm

• Artemis
  – University Nanosat 5
  – Autonomous rendezvous
  – Target & Chaser
UT Austin Satellite Program

- **Wipsat**
  - LabVIEW Embedded
  - Blackfin CPU
  - TinyBoard 28x28mm
  - CAN bus for low rate sensors
  - CMOS camera
  - GPS

www.tinyboards.com
UT Austin Satellite Program

• New Ground Station
  – 2 UHF/VHF Antennae
  – 3m Dish
  – HAM radio equipment
• Sensors & Actuators Lab
  – Control Moment Gyro
  – IMU
  – Sun Sensor
  – Image Processing
GN&C

• Guidance
  – Where am I going?

• Navigation
  – Where am I?

• Control
  – How will I get there?
Control

- Model of the real world
- Model is nonlinear
- Linearize about a nominal trajectory
- Classic linear state space model

\[
\begin{align*}
\dot{x}(t) &= A(t)x(t) + B(t)u(t) + w(t) \\
y(t) &= C(t)x(t) + D(t)u(t) + v(t)
\end{align*}
\]
Control

- Noise will be dealt with in a Stochastic (non-deterministic) manner, using Random Variables.
- Noise is assumed white (zero mean, uncorrelated), however, biased noise with color can be transformed into white noise with a bit of extra work.
Tracking and Regulation

• For a given reference input, find the control such that the controlled variables track the reference input, accounting for:
  – Plant disturbances are unpredictable
  – Plant parameters may change or may not be precisely known
  – Initial state is unknown
  – Observed variables may not give direct information about the state of the plant, and are corrupted
  – Reference input is not known a priori
Stochastic Linear Optimal* Regulator Problem

Consider the system,

\[ \dot{x}(t) = A(t)x(t) + B(t)u(t) + w(t) \]
\[ z(t) = M(t)x(t) \]

and the performance criterion,

\[
J = E \left\{ \int_{t_0}^{t_1} \left[ z^T(t)R_3(t)z(t) + u^T(t)R_2(t)u(t) \right] dt + x^T(t_1)P_1x(t_1) \right\}
\]

* Optimal ≠ Good
Stochastic Linear Optimal* Regulator Problem

When \( w(t) \) is a white noise signal, the solution to the Stochastic Linear Optimal Regulator problem is the same as the deterministic LOR problem, i.e.

\[
\begin{align*}
  u(t) &= -F(t)x(t) \\
  F(t) &= R_2^{-1}B^T(t)P(t) \\
  -\dot{P}(t) &= M^T(t)R_3(t)M(t) - P(t)B(t)R_2^{-1}(t)B^T(t)P(t) + A^T(t)P(t) + P(t)A(t) \\
  P(t_1) &= P_1
\end{align*}
\]

The presence of noise does not alter the solution, except to increase the minimal value of the performance index, \( J \).
Hardware

• Sensors
  – Gyros, accelerometers, star trackers, sun trackers, horizon sensors, etc…

• Actuators
  – Thrusters, reaction wheels, control moment gyros, torque coils/rods, gravity gradients, etc…

• Computing
  – Floating point hardware is a must!
Practical Implications

• Finite horizon (leads to mode switching)
• Stability
• Power consumption
• Nonlinear effects
  – un-modeled dynamics (damping!)
  – actuation limits
• Implementation is typically discrete time
• Momentum dumping
Simulation

• Simple Euler equations (no external torques), 3U CubeSat, no actuation model
• Notice how the control gets worse near final time - result of controller trying to get to zero
• 3 runs, different random noise
• Nominal trajectory is nadir pointing
Recent Developments

• Freescale MPC5200 based TinyBoard
  – 400 MHz w/ FPU
  – Lots of I/O
  – 35x42 mm
  – www.tinyboards.com

• German pico-reaction wheel
  – www.astrofein.com

www.tinyboards.com
Navigation

• Not enough time today.
• Hopefully, will present Nav in August at SmallSat CubeSat Workshop.
• Teaser,
  – Solution to optimal state observer (estimator) is of the same form as control solution.
  – This solution is called the Kalman Filter, but, in practice, use Extended KF, which is miles away from theory. Go Figure.
\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0
\]
\[
\frac{\partial }{\partial p_i} (\mathcal{H}) = \dot{q}_i
\]
\[
-\frac{\partial }{\partial q_i} (\mathcal{H}) = \dot{p}_i
\]