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AT AUSTIN

WHAT STARTS HERE CHANGES THE WORLD

Stochastic Attitude Control of a CubeSat

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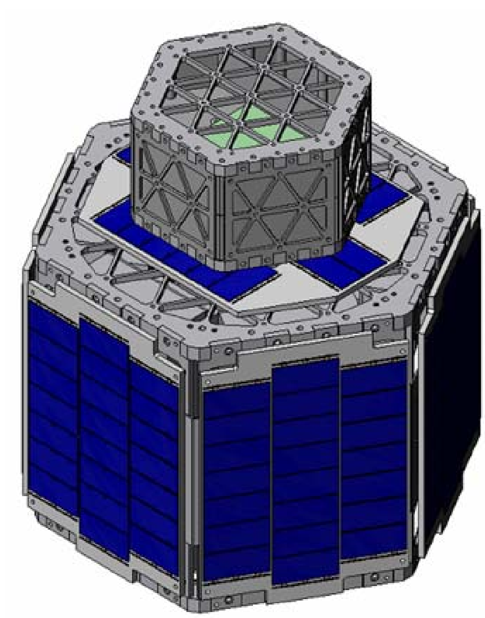
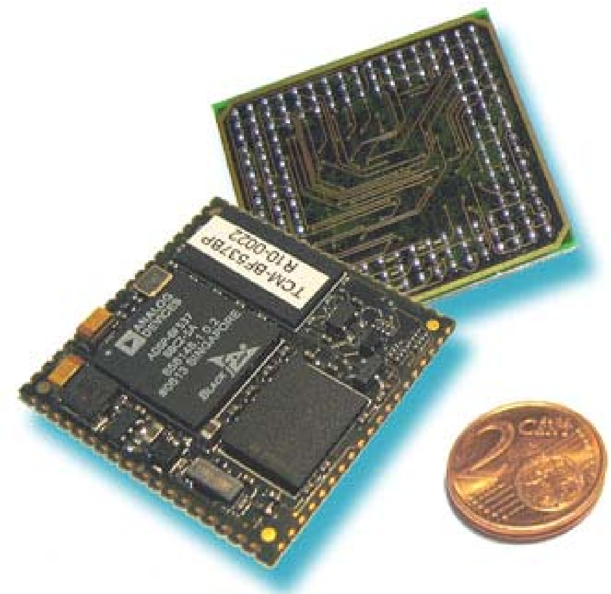
UT Austin Satellite Program

- Lonestar
  - Joint NASA / UT Austin / Texas A&M
  - Multi-year program
  - Paradigm
- Artemis
  - University Nanosat 5
  - Autonomous rendezvous
  - Target & Chaser



UT Austin Satellite Program

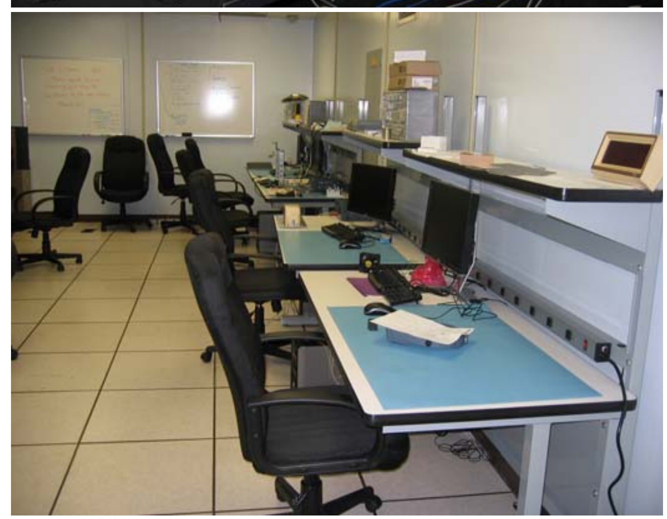
- Wipsat
  - LabVIEW Embedded
  - Blackfin CPU
  - TinyBoard 28x28mm
  - CAN bus for low rate sensors
  - CMOS camera
  - GPS



www.tinyboards.com

UT Austin Satellite Program

- New Ground Station
  - 2 UHF/VHF Antennae
  - 3m Dish
  - HAM radio equipment
- Sensors & Actuators Lab
  - Control Moment Gyro
  - IMU
  - Sun Sensor
  - Image Processing



GN&C

- Guidance
  - Where am I going?
- Navigation
  - Where am I?
- Control
  - How will I get there?

Control

- Model of the real world
- Model is nonlinear
- Linearize about a nominal trajectory
- Classic linear state space model

Process noise

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + \boxed{w(t)}$$
$$y(t) = C(t)x(t) + D(t)u(t) + \boxed{v(t)}$$

Measurement noise

Control

- Noise will be dealt with in a Stochastic (non-deterministic) manner, using Random Variables.
- Noise is assumed white (zero mean, uncorrelated), however, biased noise with color can be transformed into white noise with a bit of extra work.

Tracking and Regulation

- For a given reference input, find the control such that the controlled variables track the reference input, accounting for:
  - Plant disturbances are unpredictable
  - Plant parameters may change or may not be precisely known
  - Initial state is unknown
  - Observed variables may not give direct information about the state of the plant, and are corrupted
  - Reference input is not known a priori

Stochastic Linear Optimal* Regulator Problem

Consider the system,

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + w(t)$$
$$z(t) = M(t)x(t)$$

and the performance criterion,

$$J = E\left\{ \int_{t_0}^{t_1} [z^T(t)R_3(t)z(t) + u^T(t)R_2(t)u(t)]dt + x^T(t_1)P_1x(t_1) \right\}$$

* Optimal ≠ Good

Stochastic Linear Optimal* Regulator Problem

When w(t) is a white noise signal, the solution to the Stochastic Linear Optimal Regulator problem is the **same** as the deterministic LOR problem, i.e.

$$u(t) = -F(t)x(t)$$
$$F(t) = R_2^{-1}B^T(t)P(t)$$

$$-\dot{P}(t) = M^T(t)R_3(t)M(t) - P(t)B(t)R_2^{-1}(t)B^T(t)P(t) + A^T(t)P(t) + P(t)A(t)$$
$$P(t_1) = P_1$$

The presence of noise does not alter the solution, except to increase the minimal value of the performance index, J.

Hardware

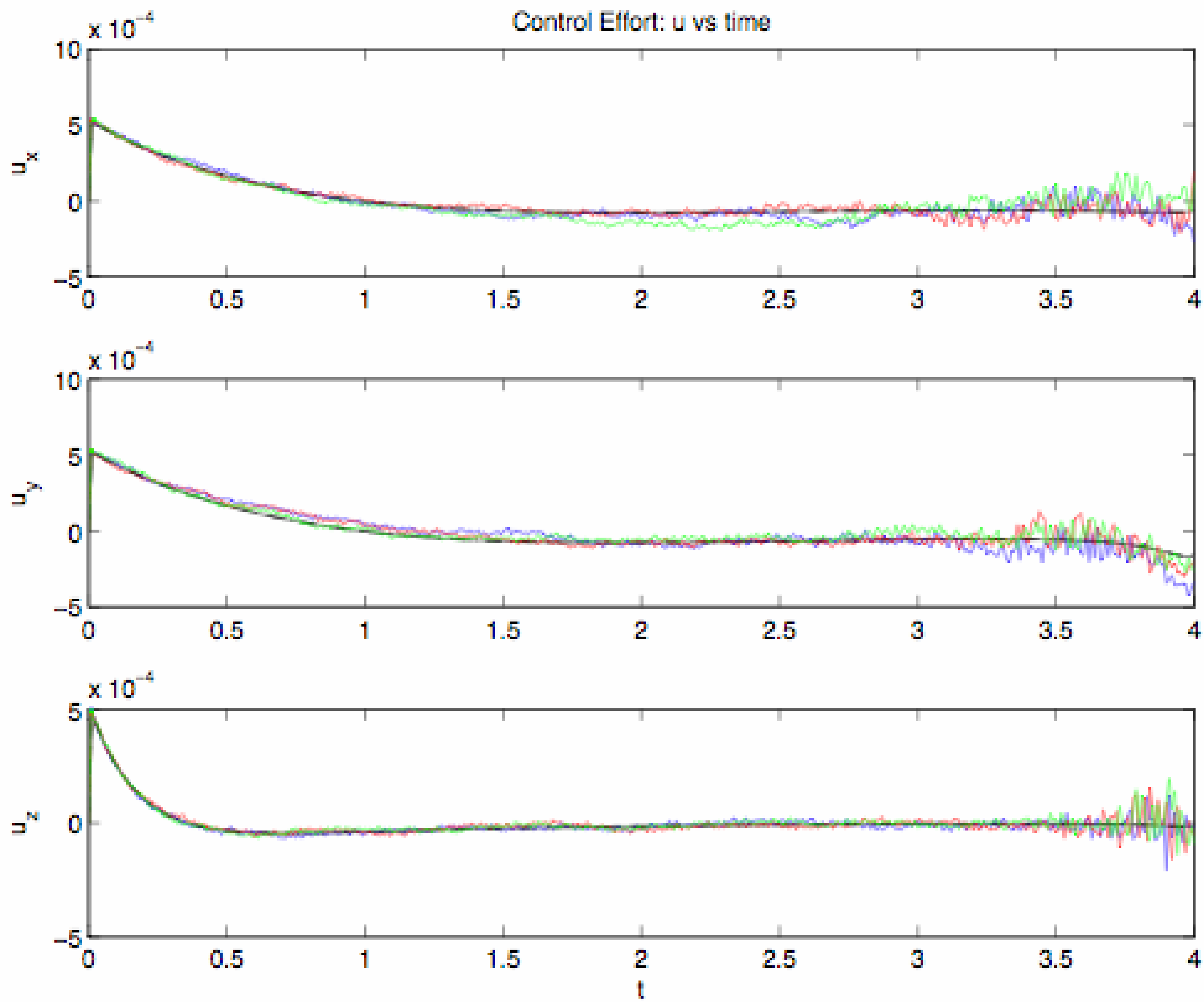
- Sensors
  - Gyros, accelerometers, star trackers, sun trackers, horizon sensors, etc…
- Actuators
  - Thrusters, reaction wheels, control moment gyros, torque coils/rods, gravity gradients, etc…
- Computing
  - Floating point hardware is a **must**!

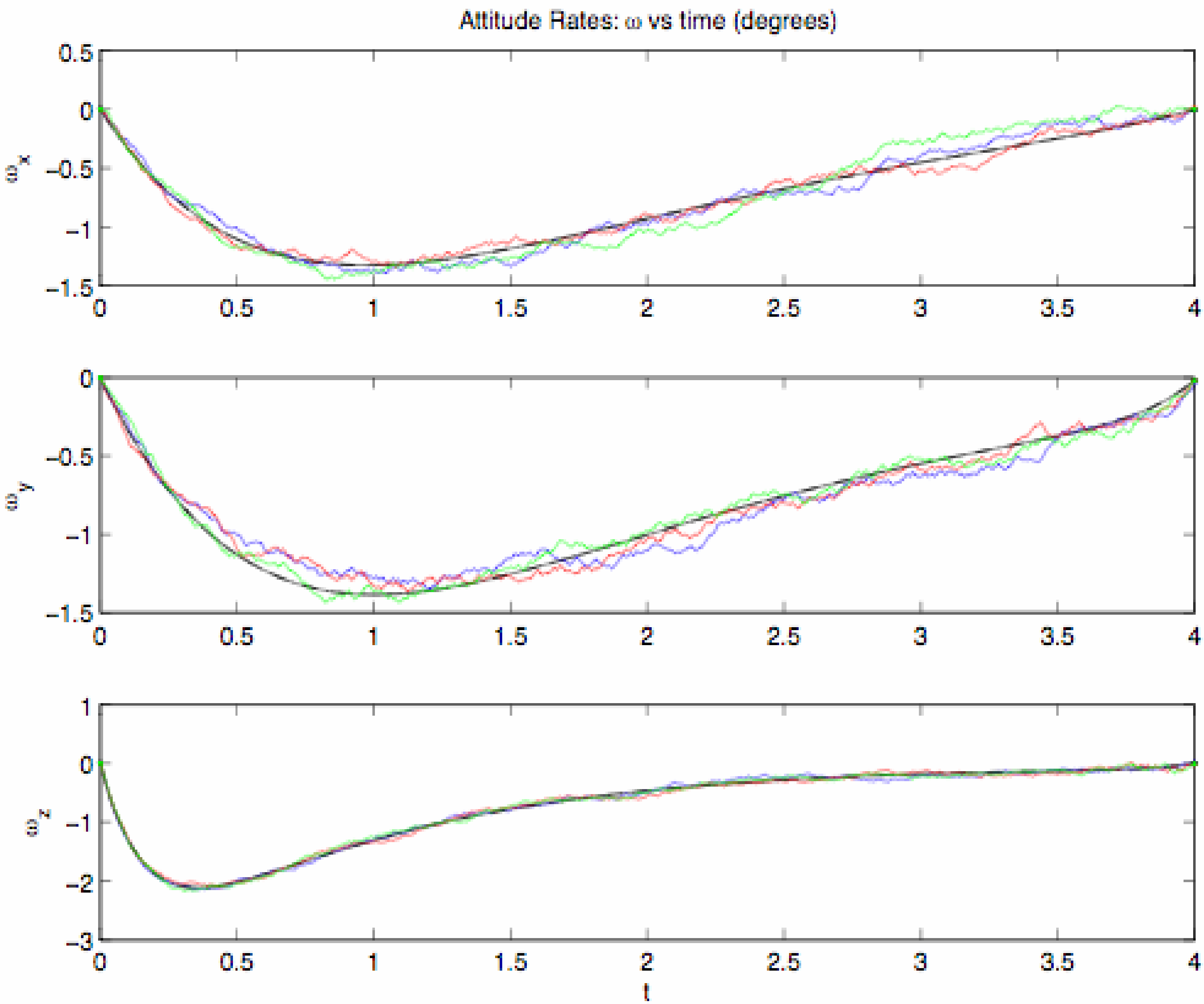
Practical Implications

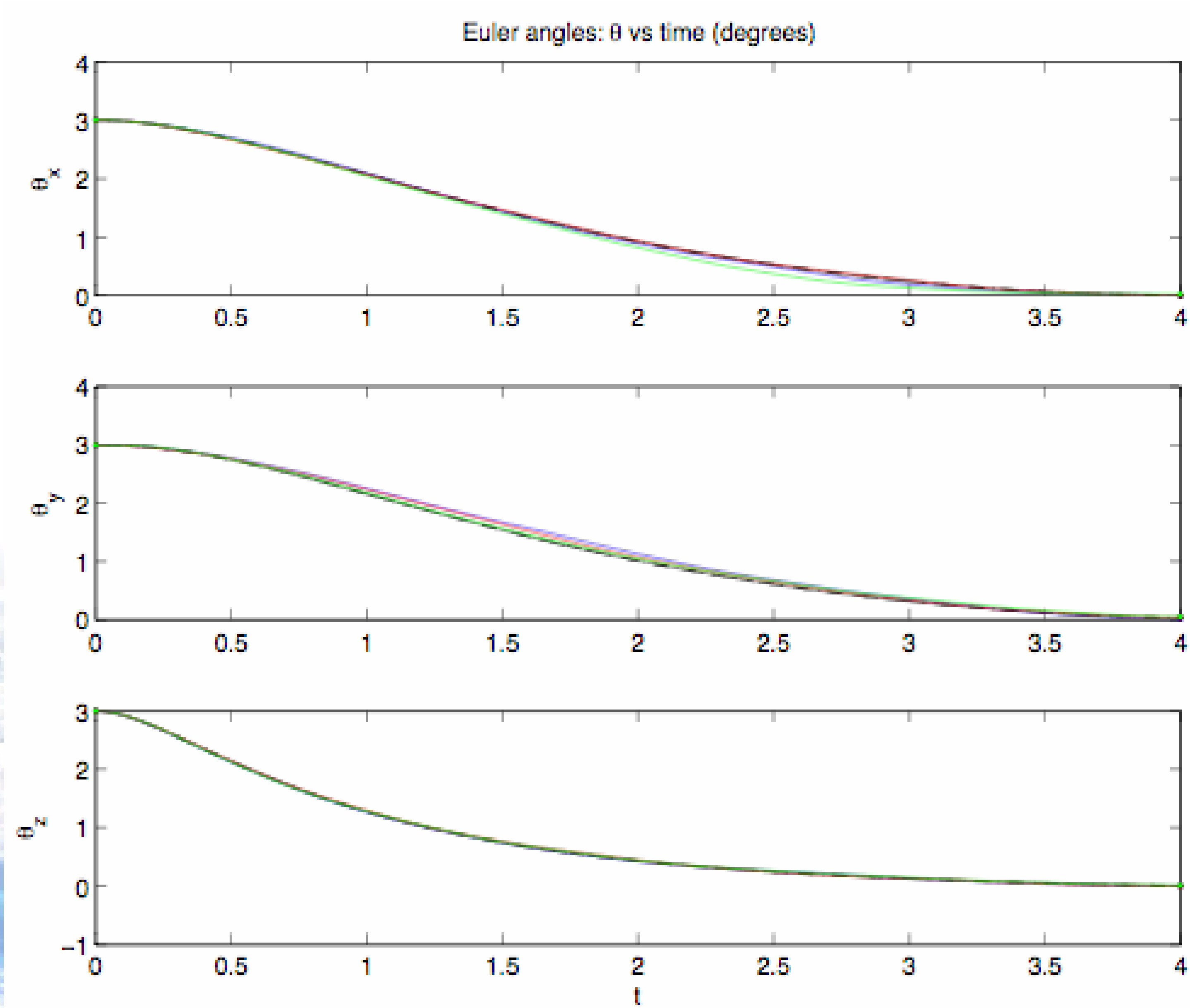
- Finite horizon (leads to mode switching)
- Stability
- Power consumption
- Nonlinear effects
  - un-modeled dynamics (damping!)
  - actuation limits
- Implementation is typically discrete time
- Momentum dumping

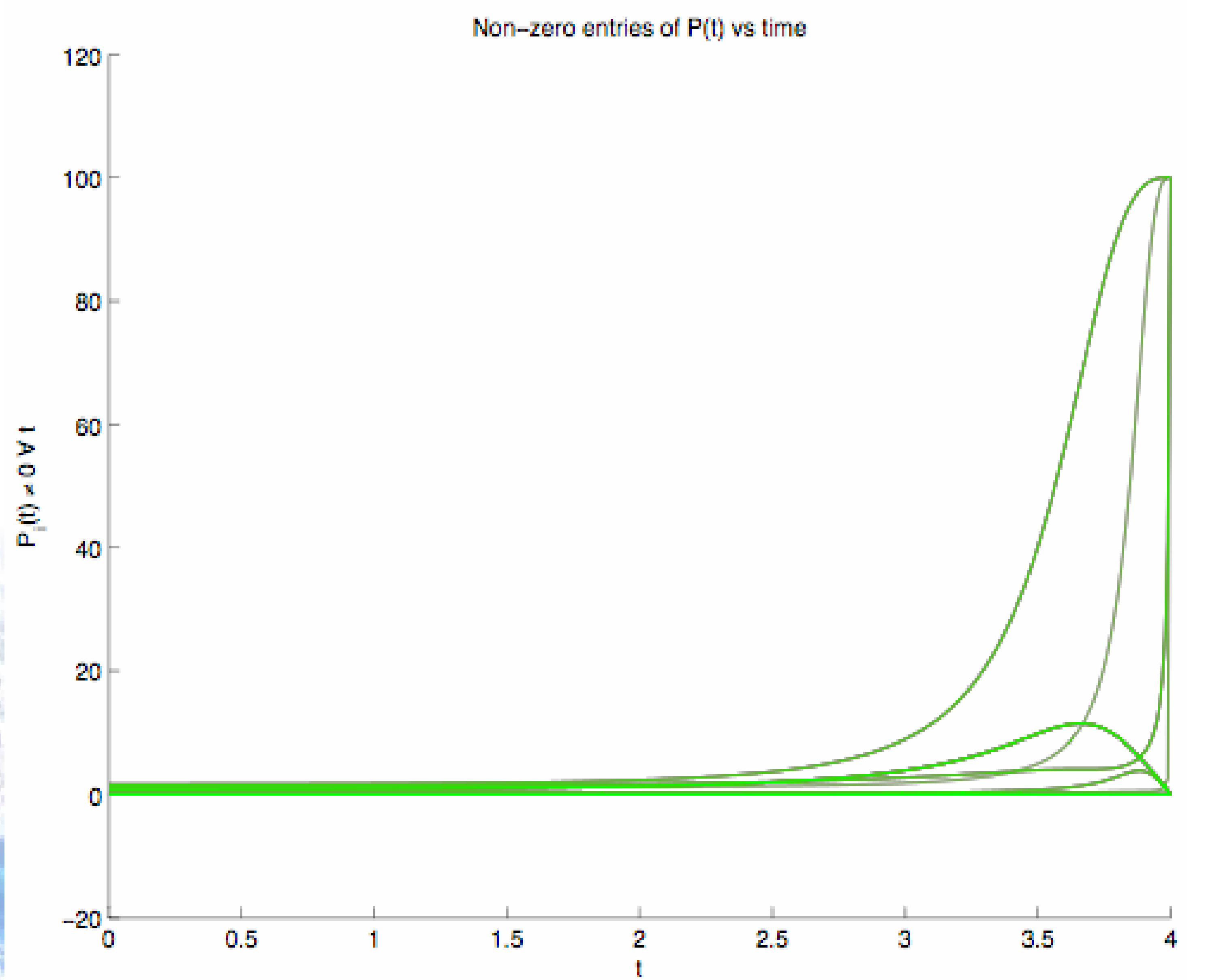
Simulation

- Simple Euler equations (no external torques), 3U CubeSat, no actuation model
- Notice how the control gets worse near final time - result of controller trying to get to zero
- 3 runs, different random noise
- Nominal trajectory is nadir pointing


Control Effort: u vs time
x 10^-4
u_x
u_y
u_z
t
10
5
0
−5
0.5
1
1.5
2
2.5
3
3.5
4


Attitude Rates: ω vs time (degrees)
ω_x
ω_y
ω_z
t


Euler angles: θ vs time (degrees)
θx
θy
θz
t
0
0.5
1
1.5
2
2.5
3
3.5
4
−1


Non-zero entries of P(t) vs time
120
100
80
60
40
20
0
-20
0
0.5
1
1.5
2
2.5
3
3.5
4
t
$P_i(t) \neq 0 \;\forall\; t$

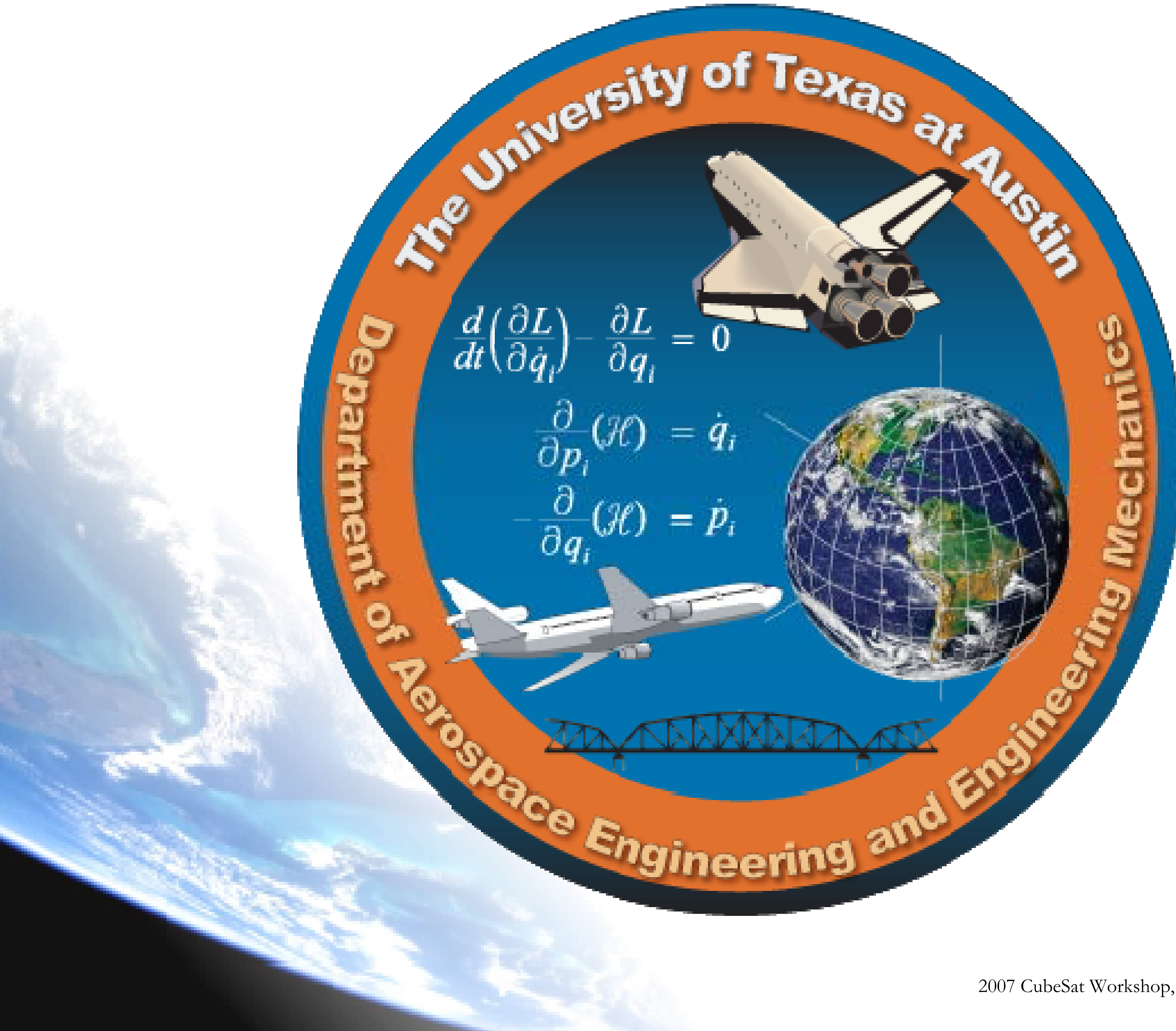
Recent Developments

- Freescale MPC5200 based TinyBoard
  - 400 MHz w/ FPU
  - Lots of I/O
  - 35x42 mm
  - www.tinyboards.com
- German pico-reaction wheel
  - www.astrofein.com

www.tinyboards.com

Navigation

- Not enough time today.
- Hopefully, will present Nav in August at SmallSat CubeSat Workshop.
- Teaser,
  - Solution to optimal state observer (estimator) is of the same form as control solution.
  - This solution is called the Kalman Filter, but, in practice, use Extended KF, which is miles away from theory. Go Figure.


The University of Texas at Austin
Department of Aerospace Engineering and Engineering Mechanics
$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$
$\frac{\partial}{\partial p_i}(\mathcal{H}) = \dot{q}_i$
$-\frac{\partial}{\partial q_i}(\mathcal{H}) = \dot{p}_i$